

SECOND TERM E-LEARNING NOTE**SUBJECT: MATHEMATICS****CLASS: JSS 3****SCHEME OF WORK**

WEEK	TOPIC
1.	Solving Simultaneous Equation by Substitution and elimination method.
2.	Solving Simultaneous Equation by graphical methods.
3.	Geometry: similar shapes
4.	Geometry continued: length, areas and volumes of similar figures
5.	Areas of plane figures
6.	Areas of plane figures continued
7.	Trigonometric ratios
8.	Angles of elevation and depression
9.	Bearing
10.	Scale Drawings.
11.	Revision

WEEK ONE**DATE-----****SIMULTANEOUS EQUATION**

Equation such as $4x + 1 = 7$, has only one solution and one unknown, thus it is called linear equation. Considering equations such as $4x + 2y = 24$ which contains two unknown quantities (x,y) it cannot be solved unless one of the variables is given or another connecting the variable is given. Hence we have two linear such as $x+y=10$; and $x-y=2$, this is known as simultaneous equation. To solve the given equations (simultaneous equation), we need to find the value of x and value of y that will satisfy both equations at the same time.

SUBSTITUTION METHOD OF SOLVING SIMULTANEOUS EQUATION

In using this method, one of the variables is made the subject of the equation. Then the value of the subject of the equation is substituted in the second equation. When the substitution is done, the equation is solved to obtain the value of one of the variables. The value is then substituted in one of the pair of equations to find the second variable.

Example: solve this pair of simultaneous equation using substitution method;

$$X+6y = -2 ; 3x+2y = 10$$

Solution:

$$X+6y = -2 \dots\dots\dots(1)$$

$$3x+2y=10 \dots\dots\dots(2)$$

From eq (1) $x = -2 - 6y$

Sub $x = -2 - 6y$ in eq(2)

$$3(-2-6y) + 2y = 10$$

$$-6-18y + 2y = 10$$

$$-16y = 10+6$$

$$-16y = 16$$

$$y = 16/-16$$

$$y = -1$$

sub $y = -1$ in eq(1)

$$x + 6y = -2$$

$$x + 6(-1) = -2$$

$$x - 6 = -2$$

Name: _____

Class: _____

$$x = -2 + 6$$

$$x = 4$$

Evaluation:

1. $x + 3y = -4$; $x + y = -10$
2. $5x - y = 35$; $3x - 2y = 14$

ELIMINATION METHOD OF SOLVING SIMULTANEOUS EQUATION

In the Elimination method, the two simultaneous equations are either added or subtracted so as to eliminate one of the variables. This is very useful to solve simultaneous equation especially when none of the coefficient of the unknown is one (1).

Example: use Elimination method to solve; $6x + 5y = 2$ and $x - 5y = 12$

Solution:

Adding:

$$\begin{array}{r} 6x + 5y = 2 \\ - | x - 5y = 12 \\ \hline 7x = 14 \\ x = 2 \end{array}$$

sub $x = 2$ in eq(1)

$$\begin{array}{l} 6x + 5y = 2 \\ 6(2) + 5y = 2 \\ 12 + 5y = 2 \\ 5y = 2 - 12 ; \quad 5y = -10 \\ y = -10/5 ; \quad y = -2 \end{array}$$

EVALUATION: simplify using Elimination method;

1. $2y - x = 10$; $y + x = 2$
2. $4p + 3q$; $3p - 5q = -10$

WORD PROBLEMS LEADING TO SIMULTANEOUS EQUATION

To solve such problems:

1. Identify the two unknowns and represent them with letters.
2. Translate the words into equations.
3. Use any convenient method to solve the two unknowns.

Example:

The difference between the ages of Audu and Ojo is 15. if the sum of their ages is 47. How old are they?

Solution: let x represents Audu's age and y represent Ojo's age.

$$\begin{array}{l} x - y = 15 \dots\dots\dots(1) \\ x + y = 47 \dots\dots\dots(2) \\ \text{Adding: } \quad \underline{2x = 62} \\ \quad \quad \quad x = 62/2 = 31 \\ \text{Substituting: } \quad x - y = 15 \\ \quad \quad \quad 31 - y = 15 \\ \quad \quad \quad -y = 15 - 31 \\ \quad \quad \quad -y = -16 \\ \quad \quad \quad y = 16 \end{array}$$

EVALUATION:

Name: _____

Class: _____

1. The cost of one orange and two apples is 24k. Two orange and three apples cost 44k. How much does each cost?
2. Six pencils and three rubbers cost N117. Five pencils and two rubbers cost N92. How much does each cost?

Reading Assignment

Essential mathematics by A.J.S OLUWASANMI Pg 148-152

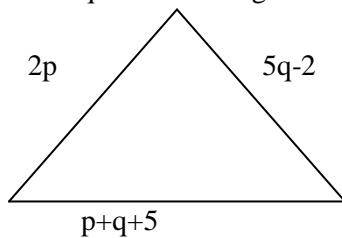
Exam focus for J.S.S CE Pg 220-222

WEEKEND ASSIGNMENT

1. If $X=2$, the value of y in $y=8-4x$ is A. 1 B. 2 C.3
2. If the equation $y = mx+c$ is satisfied by $x=1, y=5$ and $c=0$, the value of m is A. 3 B. 5 C. 2
3. the solution of x if $y=5x+2$; and $x+2y=15$ is A. 1 B. 2 C. 5
4. The sum of two numbers is 18 and their difference is 12. Find the two numbers from the above question.A.6&10 B.15&3 C. 10&3
5. What is the product of the two numbers A. 60 B.45 C. 30

THEORY

1. The sum of the ages of a man and his wife is 73yrs. Eight years ago the husband was twice as old as the wife. How old are they now?
2. The below is an equilateral triangle with the dimensions shown:



Find

- (a) The value of p and q
- (b) The perimeter of the triangle in meters
- (c) The area of the triangle to 3.s.f.g

WEEK TWO

DATE.....

GRAPHICAL METHOD OF SOLVING SIMULTANEOUS EQUATION

Expressions in x written as $ax+b$ where a and b are constants (which can be any number) are known as linear expression. Thus we can draw a graph representing the above expression by equating it to y . to draw a linear graph we select suitable values of x so as to calculate the values of corresponding y . hence to draw a simultaneous equation, we make y the subject in each of the equation. Then find the values of the corresponding y with the selected suitable values of x .

Steps in using graphical method

1. Make y the subject in each equation.
2. Draw a table of values for each of the linear equations; taking a range of values.
3. On a graph paper label the x -axis and y -axis according to the table drawn in step 2(two) above.
4. Plot these values and join the points for each of the table of values.
5. Take note of the point of intersection of the two lines. At this point trace it to both y and x axes. The values are the only pair of values that satisfy both simultaneous equations.

Example:

Solve graphical the simultaneous equation below

Name: _____

Class: _____

$$X + y = 6; 3x - y = 12.$$

Solution:

1. From eq(1) $y = 6 - x$
From eq(2) $y = 3x + 2$
2. Draw the table of values of the equations taking a range of values ie

Table for $y = 6 - x$

X	-1	0	1	2	3
y	7	6	5	4	3

Guiding equation: $Y = 6 - x$

- (i) when $x = -1$, $Y = 6 - (-1)$, $Y = 6 + 1$, $Y = 7$
- (ii) when $x = 0$, $Y = 6 - 0$, $Y = 6$
- (iii) when $x = 1$, $Y = 6 - 1$, $Y = 5$
- (iv) when $x = 2$, $Y = 6 - 2$, $Y = 4$
- (v) when $x = 3$, $Y = 6 - 3$, $Y = 3$

Table for $Y = 3x + 2$

Guiding equation: $Y = 3x + 2$

X	-1	0	1	2	3
y	-1	2	5	8	11

- (i) when $x = -1$, $Y = 3(-1) + 2$; $Y = -3 + 2$; $Y = -1$
- (ii) when $x = 0$, $Y = 3(0) + 2$; $Y = 0 + 2 = 2$
- (iii) when $x = 1$, $Y = 5$
- (iv) when $x = 2$, $Y = 8$
- (v) when $x = 3$, $Y = 3(3) + 2$; $Y = 9 + 2 = 11$

EVALUATION

1. What is a linear equation?
2. Which of these equations are linear? A. $a + b$ B. $a^2 + b = 12$ C. $x - 1 = 2$
3. What is the first step in drawing graph?

Further exercises on the use of graph to solve simultaneous equation

In making of table of values for points to be plotted, x is called independent variable while y is the dependent variable.

The point where the variable crosses an axis is called an intercept.

Example:

Draw the graph of the given pair of the equation $2x - y = 3$, $x + y = 6$ and show the point of intercept of the lines on the y -axis.

Solution:

- (i) Make the y subject in each equation. I.e. $Y = 2x - 3$; $Y = 6 - x$
- (ii) Make a table of values for each equation with ranges of $Y = 2x - 3$

X	-1	0	1	2	3
y	-5	-3	-1	1	3

- (iii) Make the table of value for $Y = 6 - x$

X	-1	0	1	2	3
y	7	6	5	4	3

From the graph the points of intercept are -3 and -6 .

EVALUATION:

Solve graphically the below simultaneous equation:

(a) $Y - x = -4$; $Y + 3x = 12$

(b) $8c + 3d = 1$; $4c + 5d = 9$

READING ASSIGNMENT:

Essential mathematics for J.S.S 3 Pg 146-147

Exam focus for J.S.S CE Pg 218-219

WEEKEND ASSIGNMENT

- How many variables do we have in $x + y + z - 6 = 128$ A. 2 B. 1 C. 3
- What axes are used in the plotting of graph? A. P&Q B. X&Y C. P&Y
- when given that $Y = 2x - 1$, what is the y if $x = -1$ A. -3 B. -2 C. 1
- Given that coordinates at the point of intersection of a drawn graph is $(-1, 3)$, the values of y is ... A. -1 B. 3 C. 2
- Make n the subject in $9m - 4n = -36$ n = A. $\frac{-36 + 9m}{4}$ B. $\frac{9m + 36}{4}$ C. $\frac{36 - 9m}{4}$

THEORY

Solve the following pairs of simultaneous equations graphically:

1. $2x + y = 8$; $x + y = 5$ 2. $6x + y = 12$; $x - y = 9$

WEEK THREE**TOPIC: Geometry**

Similar Triangles

One of the following conditions is sufficient to show that two triangles are similar.

- If two angle of one triangle are equal to two angles of the other.
- If two pairs of sides are in the same ratio and their included angles are the same.
- If the ratios of the corresponding sides are equal.

Example

Show that $\triangle ABC$ and $\triangle XYZ$ shown below are similar and hence find sides AB and XZ.

Solution

In $\triangle ABC$:

$$\begin{aligned} \angle A &= 180 - (32 + 38) \\ &= 110^\circ \end{aligned}$$

Similarly, in $\triangle XYZ$

$$\begin{aligned} \angle Z &= 180^\circ - (110^\circ + 32^\circ) \\ &= 38^\circ \end{aligned}$$

$$\angle A = \angle X = 110^\circ, \angle B = \angle Y = 32^\circ \text{ and}$$

$$\angle C = \angle Z = 38^\circ$$

Therefore, Triangles ABC and XYZ are similar because they are equiangular

Hence: $\frac{AB}{XY} = \frac{AC}{ZY} = \frac{BC}{YZ}$

Substituting the given sides:

$$\frac{AB}{2} = \frac{25}{XZ} = \frac{35}{7}$$

$$\text{Hence: } \frac{AB}{2} = \frac{35}{7} \text{ and } \frac{25}{XZ} = \frac{35}{7}$$

$$\frac{AB}{2} = \frac{35}{7} \text{ and } \frac{25}{XZ} = \frac{35}{7}$$

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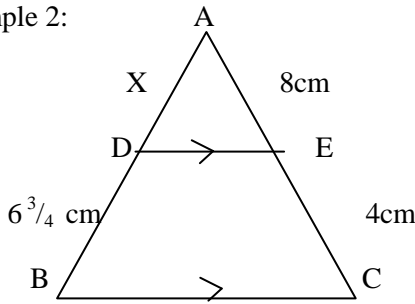
Class: _____

$$7AB = 2 \times 35, \quad 35XZ = 25 \times 7$$

$$AB = \frac{2 \times 35}{7} \text{ and } XZ = \frac{25 \times 7}{35}$$

$$AB = 2 \times 35 \text{ and } XZ = 5\text{cm}$$

Example 2:



In the diagram shown above, line DE and BC are parallel. $AE = 8\text{cm}$, $EC = 4\text{cm}$, $BD = 6 \frac{3}{4} \text{cm}$

(a) Show that triangle ABC and ADE are similar

(b) Calculate AD

Solution

(a) $\angle A$ is common to both triangle $\triangle ABC$ and $\triangle ADE$

$$\angle ACB = \angle AED$$

Triangle ADE and ABC are similar because they are equiangular

(b) Let side AD be Xcm

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{X}{X + 27/4} = \frac{8}{12}$$

$$12X - 8x = 54$$

$$4x \frac{54}{4} = 13.5\text{cm}$$

$$\text{So } X = AD = 13.5\text{cm}$$

READING ASSIGNMENT

Essential Mathematics Page 161

Exercise 18.3; 1-11

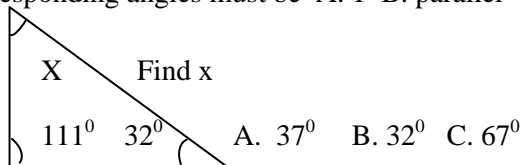
WEEKEND ASSIGNMENT

Objectives

1. Similar triangles are _____ A. Equiparallel B. Equiangular C. Parallel

2. Ratio of corresponding angles must be A. 1 B. parallel C. constant

3.

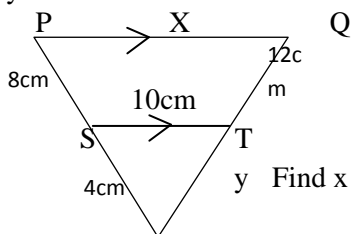


4. If $AB = 6\text{cm}$ and $AD = 8\text{cm}$. What is the ratio of corresponding sides A. $\frac{3}{4}$ B. $\frac{5}{4}$ C. $\frac{4}{5}$

5. The sum of angles in a triangle is ____ A. 360° B. 270° C. 180°

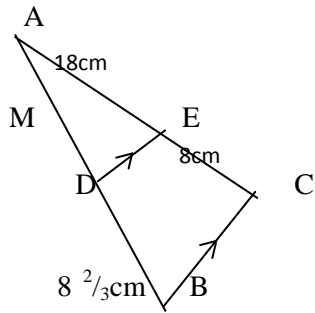
Theory

1.



Y

2.



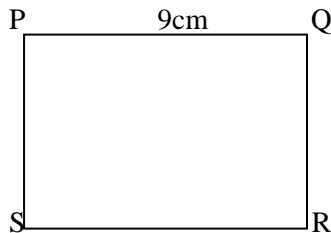
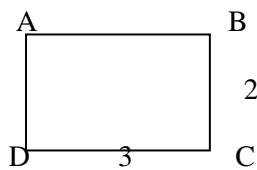
Find m

WEEK FOUR

TOPIC: GEOMETRY CONT'D

Areas and Volumes of similar shaped

Scale Factor (Length Ratio)



6cm

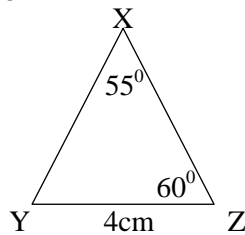
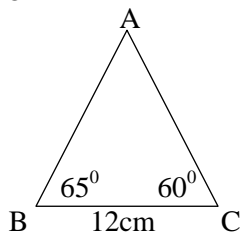
$$\text{Scale factor} = \frac{9}{3} \text{ or } \frac{6}{2} = \frac{3}{1}$$

$$\text{Area factor} = (\text{Scale factor})^2 = \left(\frac{3}{1}\right)^2 = 9/1 \text{ or } 9.1$$

If the ratio of the length is X : Y
Then the ratio of the area is X² : Y²

Example 1:

In the diagram below, if the area of triangle ABC is 48cm², find the area of triangle XYZ to 3s.f



Solution

$$\text{In } \triangle ABC, \angle A = 180 - (65 + 60) = 55$$

$$\text{In } \triangle XYZ, \angle Y = 180 - (55 - 60) = 65^{\circ}$$

Since then corresponding angles are equal, $\triangle ABC$ and $\triangle XYZ$ are similar

$$\text{Scale factor} = \frac{4}{12} = \frac{1}{3}$$

$$\text{Area factor} = \left(\frac{1}{3}\right)^2 = 1/9$$

$$\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} = \frac{1}{9}$$

$$\frac{\text{Area of } \triangle XYZ}{48} = \frac{1}{9}$$

$$\text{Area of } \triangle XYZ = \frac{48}{9} = 5.333\text{cm}^2$$

$$\text{Area of } \triangle XYZ = 5.3 \text{ cm}^2 \text{ to 2 s.f.}$$

Volume of Similar Shapes

$$\text{Volume of factor} = (\text{Scale factor})^3$$

If the ratio of the length is X : Y

Then the ratio of the volume is $X^3 : Y^3$

Example 2: The ratio of a cylinder of volume 2970cm^3 is 30mm. find the volume of a similar cylinder of radius 40mm.

Solution

$$\text{Scale factor} = \frac{40\text{mm}}{30\text{mm}} = \frac{4}{3}$$

$$\text{Volume ratio} = \frac{4^3}{3^3}$$

$$\frac{V}{2970} = \frac{4^3}{3^3}$$

Cross-multiply

$$V = 7040\text{cm}^3$$

The volume of the cylinder is 7040 cm^3

READING ASSIGNMENT

Essential Mathematics page 182

Exercise 19.2, 1-14

Exercise 19.1 ; 1- 19

WEEKEND ASSIGNMENT

1. An object has an area of 3.2cm^2 on a map is 1: 1000, find the actual area of the object in m^2 .
2. Two similar circles have corresponding radius in the ratio 4:9. What is the ratio of their area.

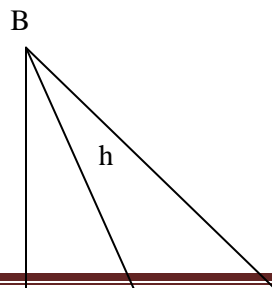
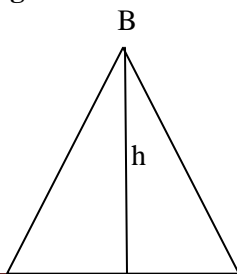
Objectives

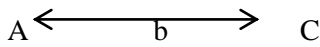
1. If the scale factor is 2:3, what is the area factor A. 4: 6 b. 6: 4 C. 4:9
2. If the area factor is 4:9, what is the volume factor A. 8: 27 B. 64:81 C. 27:8
3. Area factor is the _____ of scale factor A. cube B. square C. root
4. The corresponding altitudes of two similar triangles are 8cm and 5cm. Find the ration of their areas? A. 25:16 B. 64:25 C. 64:10
5. If $\frac{2}{3} = \frac{x}{9}$ find x
A. 27 B. 18 C. 6

WEEK FIVE AND SIX

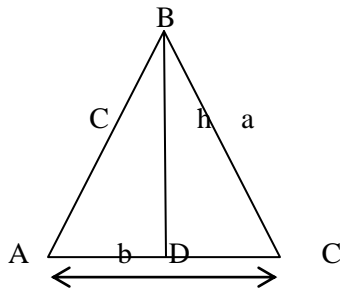
TOPIC: Area of Plane Figures

Area of Triangle





Area of $\Delta ABC = \frac{1}{2}$ base X height = $\frac{1}{2} bh$



$\sin A = \frac{h}{c}$

$h = c \sin A$

Area of $\Delta ABC = \frac{1}{2} bc \sin A$

Example 1:

Find the area of triangle PQR if sides PQ = 6cm, PR = 8cm and QR = 10cm

Solution

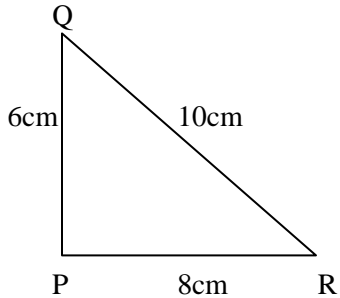
First, we need to show that ΔPQR is a right angled triangle

$PQ^2 + PR^2 = QR^2$

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

ΔPQR is a right – angled triangle since 6, 8 and 10 Pythagoras triples

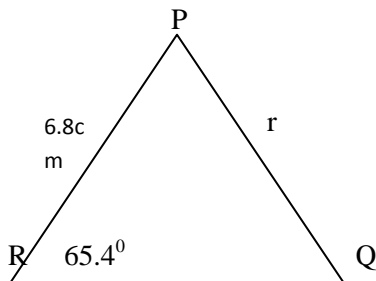


Area of $\Delta PQR = \frac{1}{2} \times 8 \times 6 = 24\text{cm}^2$

Example 2:

Calculate the area of triangle PQR correct to 3 significant figures if p = 8.5cm, q = 6.8cm and R = 65.4°

Solution



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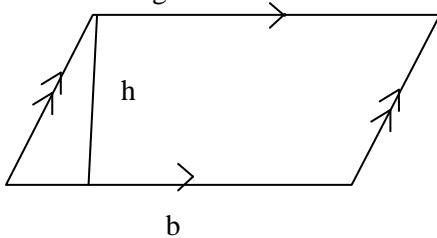
Class: _____

P 8.5cm

$$\begin{aligned}\text{Area of } \triangle PQR &= \frac{1}{2} pq \sin 65.4 \\ &= \frac{1}{2} \times 8.5 \times 6.8 \times \sin 65.4 \\ &= 26.276\text{cm}^2\end{aligned}$$

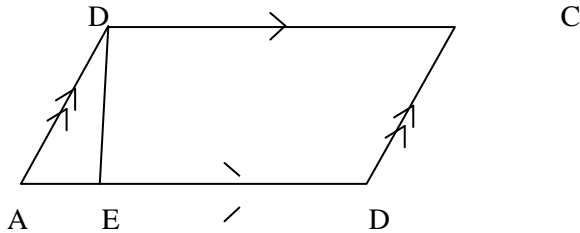
$$\text{Area} = 26.3\text{cm}^2 \text{ to 3 s.f}$$

Area of Parallelograms



$$\text{Area of parallelogram} = \text{base} \times \text{height} = bh$$

Consider the parallelogram



In general,

Area of parallelogram = product of adjacent sides x the size of angle between the two sides

Example 3

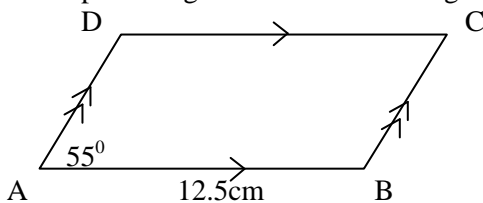
Find the area of parallelogram with base 12cm and height 7cm

Solution

$$\begin{aligned}\text{Area of parallelogram} &= \text{base} \times \text{height} \\ &= 12\text{cm} \times 7\text{cm} \\ &= 84\text{cm}^2\end{aligned}$$

Example 4:

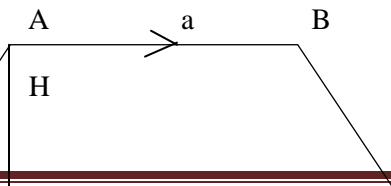
Find the area of parallelogram shown in the diagram below



$$\begin{aligned}\text{Area of } \triangle ABC &= 12.5 \times 8.4 \times \sin 55 \\ &= 105 \times 0.8192 \\ &= 86.061\text{cm}^2\end{aligned}$$

The area of $\triangle ABC = 86.061\text{cm}^2$ to 1 d.p

Trapezium



Name: _____

Class: _____

D b > C

Area of trapezium = $\frac{1}{2}$ of (sum of parallel sides) x height
Area of trapezium = $\frac{1}{2}(a + b)h$

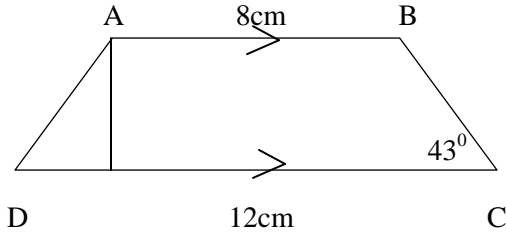
Rhombus

Area of Rhombus = base x height
= bh

OR Area of Rhombus = $\frac{1}{2}$ of product of diagonals

Example 3:

Find the area of trapezium ABCD shown below if AB = 8cm, BC = 6cm, DC = 12cm and angle BCD = 43°



Solution:

Area of trapezium ABCD = $\frac{1}{2}(AB + DC)h$

$\sin 43^\circ = \frac{h}{6}$

$h = 6 \times \sin 43^\circ$

$= 6 \times 0.6821 = 4.092$

Area of ABCD = $\frac{1}{2}(8 + 12) \times 4.092$
 $= \frac{1}{2} \times 20 \times 4.092 = 40.92 \text{cm}^2$

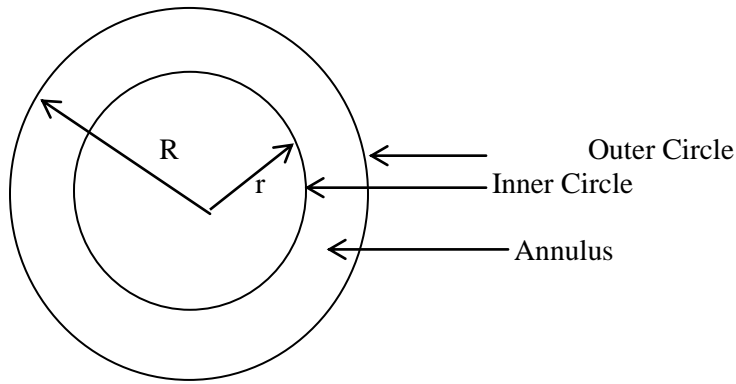
The area of trapezium ABCD = 41cm^2 to the nearest cm^2 .

Area of Circles

Area of circle = πr^2

where $r = \frac{d}{2}$

Area of annulus



$$\begin{aligned} \text{Area of annulus} &= \pi R^2 - \pi r^2 \\ &= A = \pi(R^2 - r^2) \end{aligned}$$

READING ASSIGNMENT

Essential Mathematics page 220

Ex 21.5 ; 1 – 23

WEEKEND ASSIGNMENT

- The area of a parallelogram is given as 108cm^2 . The height of the parallelogram is 9cm, find the base of the parallelogram A. 13cm B. 9cm C. 12cm
- Find the area of a rhombus of side 20mm and height 10mm A. 20mm^2 B. 200mm^2 C. 300mm^2
- Find the area of a circle of diameter 35cm
- The area of a circle is 1386cm^2 . Find the diameter of the circle A. 21cm B. 42cm C. 82cm
- A sector of a circle of radius 8cm has an area of 120° at the centre. Find its perimeter A. 33 B. 34 C. 36

THEORY

- A circle has an area of $144\pi\text{cm}^2$. Calculate the circumference of the circle, leaving your answer in terms of π
- Calculate the area of an annulus, which has an external diameter of 25cm and internal diameter of 15cm.

WEEK SEVEN

DATE-----

TRIGONOMETRICAL RATIO

Trigonometrical ratio is a ratio of the lengths of two sides of a right-angle triangle. The three trigonometrical ratios are sine (sin) cosine (cos) and tangent (tan). The word tri- means three, thus trigonometrical ratio deals with three sided figure (triangle).

In a right-angled triangle, the longest side is called the **hypotenuse** (opp the right angle), the side adjacent (next) to the given angle is called the **Adjacent** while the side opposite to the given angle is called the **opposite**.

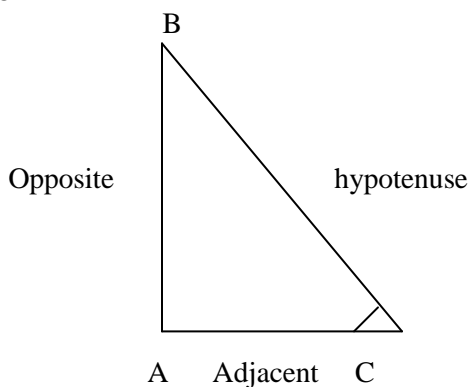


Fig 1

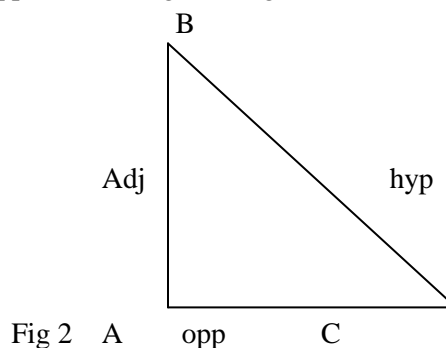


Fig 2

Name: _____

Class: _____

Note: To be able to know the ratio easily take note of the acronym SOHCAHTOA. Where S stands for sine, C stands for cosine, and T for tangent.

Degree and Minutes

Angles are often measured to the nearest degree. In some state, degree may be subdivided into minutes and.

Note:

$1^{\circ} = 60$ minutes. This is written as $60'$.

To change from minutes to degree, we divide the number by 60.

Example: convert 10° to minutes

Solution: $10 \times 60 = 600$ mins

EVALUATION:

1. Convert the following to minutes: A. 16° B. 50°
2. Rewrite and give your answer in degree to 1.dp A. $46^{\circ} 15'$ B. $39^{\circ} 25'$ C. $140^{\circ} 4'$

SINE OF ANGLE

In a right-angle triangle, the ratio of opposite to hypotenuse is defined as the sine of the angle under consideration.

From fig 1, $\sin \phi = AB/BC$. The ratio does not depend on the size of the triangle but depends only on the size of the angle (ϕ).

To find sine of the angles, we use of either calculator or the sine table. In use of sine table, since the sine of angle increases as the angle increases, thus the differences will be added.

EVALUATION

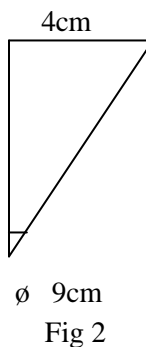
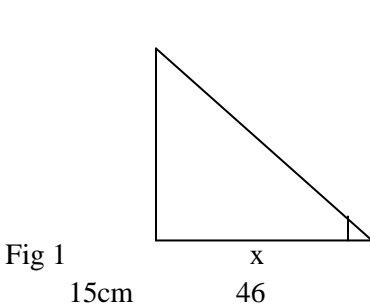
use mathematical table to find

- (i) $\sin 43$
- (ii) $\sin 14.58$
- (iii) $\sin 30.6$

USE OF SINE IN SOLVING TRIANGLES

Example:

Find the marked side or the angle in each of the following. Give your answer to 2.s.f.g.



Solution:

From fig 1

$$\sin 46 = 15/x$$

$$X = 15/\sin 46$$

$$X = 15/0.7193$$

$$X = 20.85, \quad X = 21 \text{ (2.s.f.g)}$$

From Fig 2

$$\sin \phi = \text{opp/hyp}$$

$$\sin \phi = 4\text{cm}/9\text{cm}$$

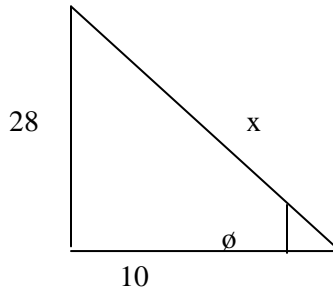
$$\sin \phi = 0.444$$

$$\phi = \sin^{-1} 0.4444$$

$$\phi = 26.49, \quad \phi = 26 \text{ (2.s.f.g)}$$

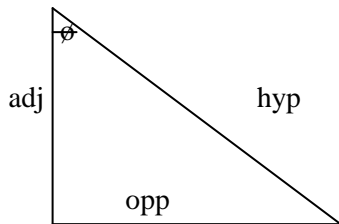
EVALUATION

What is value of X and ϕ in the below triangle

**COSINE OF ANGLES**

In a right angled triangle, the ratio of adj/hyp is defined as the cosine of angle under consideration.

Using diagram:

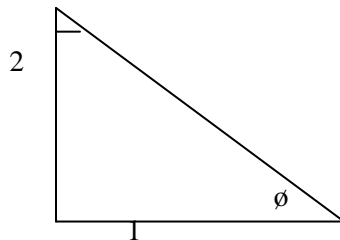


$$\text{Thus } \Theta = \text{AB/AC}$$

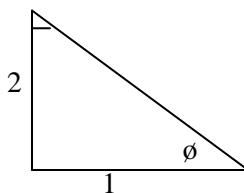
This value of the ratio does not depend on the size of triangle but on the size of angle.

CALCULATIONS OF COSINE OF ANGLES

Find the unknown side or angle in the below triangles



solution:

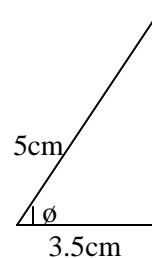
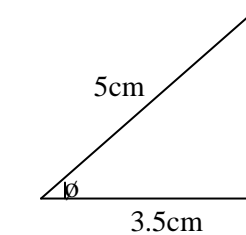


$$\cos \phi = \text{Adj/hyp}$$

$$\cos \phi = 1/2$$

$$\phi = \cos^{-1} 0.5$$

$$\phi = 60^\circ$$



$$\cos \phi = \text{adj/hyp}$$

$$\cos \phi = 3.5\text{cm} / 5\text{cm}$$

$$\phi = \cos^{-1} 0.7$$

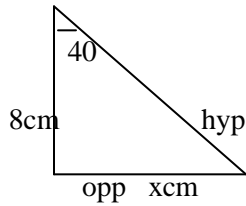
$$\phi = 45.67$$

TANGENT OF ANGLES

The tangent of **any angle** is the **ratio opp/adjacent**. In short form, $\tan \Theta = \text{opp/adj}$

CALCULATING TANGENT OF TRIANGLES

Examples: find the side of the triangle marked x . correct to 2 S.F.G in the figure below.



solution:

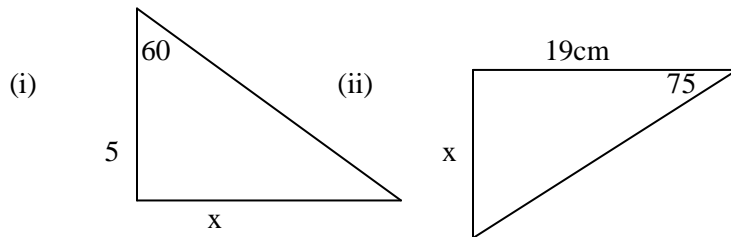
$$\tan \theta = \text{opp/adj} = x\text{cm}/8\text{cm}$$

$$\tan \theta = x/8, \theta = \tan^{-1} x/8$$

$$\theta = 0.8391x/8$$

$$\theta = 6.7128; \theta = 6.7(2 \text{ s.f.g})$$

EVALUATION: calculate the side of the triangle marked x

**READING ASSIGNMENT**

Essential mathematics for J.S.S 3 Pg 101-116

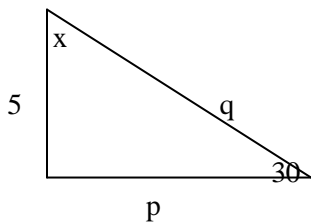
Exam Focus for J.S.C.E. for J.S.S 3 Pg 224-235

WEEKEND ASSIGNMENT

- Convert 32.4° to degree and minutes. A. $32^{\circ} 42'$ B. $32^{\circ} 44'$ C. $32^{\circ} 24'$
- $\cos 60$ is equal to ----- A. 0.5 B. 0.49 C. $1/25$

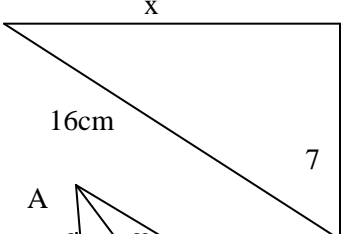
Calculate the side marked P,Q, and α

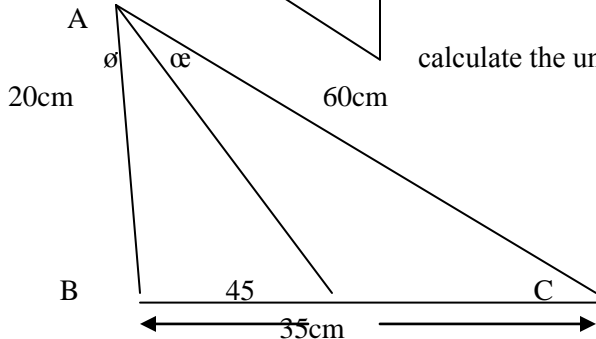
Draw:



- the value of P is A. 9.6 B. 8.7 C. 10
- the value of q is --- A. 10 B. 8 C. 13
- the value of α is ---A. 45 B.60 C. 30

THEORY

1.  calculate x

2.  calculate the unknown angles.

WEEK EIGHT

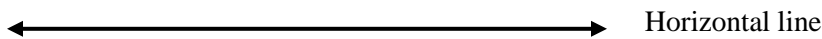
Date:.....

TOPIC: ANGLES OF ELEVATION AND DEPRESSION

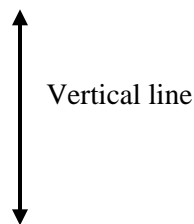
- CONTENT:**
- i) Horizontal and vertical lines
 - ii) Angles of elevation
 - iii) Measuring angles of elevation and depression.

Horizontal and Vertical Lines

Horizontal lines are lines that are parallel to the surface of the earth. For example, the surface of a liquid in a container, floor of a classroom, etc. See the diagram below:



Vertical lines are lines that are perpendicular to the horizontal surface, e.g. wall of a classroom, a swing pendulum, etc.



Evaluation:

Say whether the following are horizontal or vertical or neither.

- a) A table top
- b) A door
- c) A table leg
- d) Top edge of a wall.

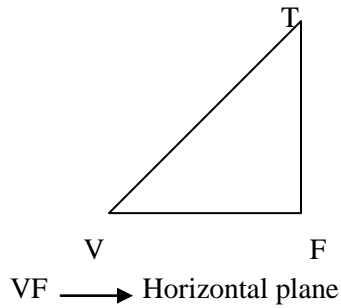
READING ASSIGNMENT

NGM Bk 2 Chapter 20, Page 165

Essential Mathematics for JSS Bk 2, Chapter 17, Pg 173

Angles of Elevation

The angle of elevation of an object for a given point above the surface of the earth is the angle formed between the horizontal plane and the view point of the object. See the diagram below. T



V = View point, T = Top where the object is, F = Foot of the vertical plane
e = angle of elevation

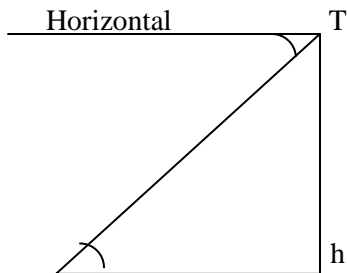
Reference:

NGM Bk 2 Chapter 20, Page 165

Essential Mathematics for JSS Bk 2, Chapter 17, Pg 173

Angle of Depression

The angle of depression of an object from a given point T is angle from the horizontal line above the earth's surface and the vertical surface.



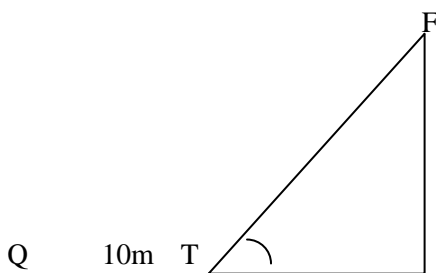
Thus, the angle of elevation is equal in size to the angle of depression. (alternate angles are equal i.e. $d = e$)

Measuring Angles of Elevation and Depression

When constructing angles of elevation and depression, the use of scale drawings is necessary in order to have effective construction of angles

Worked Examples

1. Consider the diagram below; find the height of the flagpole to the nearest metre using suitable scale.



Solution

By construction, choose a scale of 1cm represent 2m.

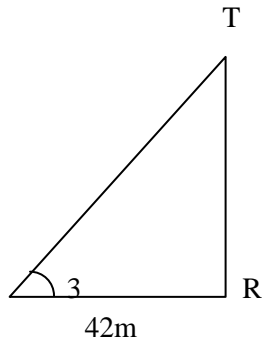
The height of the flagpole PT = 3cm, converted to m, will give $2 \times 3 = 6$.

Name: _____

Class: _____

Example 2: The angle of elevation of the top of a tower to a point 42m away from its base on level ground is 36° , find the height of the tower.

Solution



By construction, using a suitable scale of 1cm represented by 6m, then $PR = 42/6 = 7\text{cm}$.

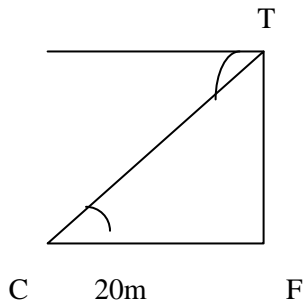
The length $TR = 5.0\text{cm}$. Converting back to metre, we have;

Length $TR = 5 \times 6 = 30\text{m}$

Example 3: From the top of a building 20m high, the angle of depression of a car is 45° , find the distance of the car from the foot of the building.

Solution

Rough sketch:



T = Top of the building, C = Car, F = Foot of the building

CF is the distance of the car from the foot of the building.

Since angle of depression equals angle of elevation;

By construction, using a suitable scale of 1cm represents 5m

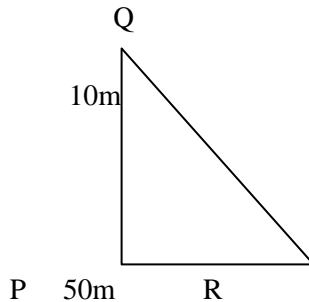
For 20m, we have $20/5 = 4\text{cm}$

Length $CF = 4\text{cm}$

By conversion, length $CF = 4 \times 5 = 20\text{m}$.

Evaluation:

1. A tower PQ is 10m high, if the distance from point R to P is 50m on the ground, find the angle of elevation of Q from R.



2. From the top of a cliff of 200m high, Dele observes that the angle of depression of a boat at

Name: _____

Class: _____

sea is 35° , find the distance between the boat and the foot of the cliff.

READING ASSIGNMENT

NGM Bk 2 Chapter 20, Page 166 - 169

Essential Mathematics for JSS Bk 2, Chapter 17, Pg 176 – 177.

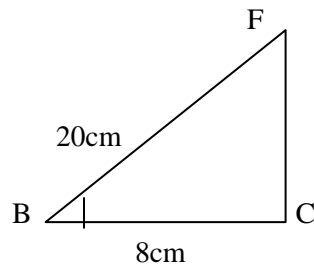
WEEKEND ASSIGNMENT

Objective

1. Calculate the size of the fourth angle if three angles of a quadrilateral are 65° , 115° , and 125° respectively. a) 35°
b) 55° c) 45° d) 75°
2. Calculate the number of sides of a regular polygon whose total angle is 1080° . a) 4 b) 6 c) 8 d) 10
3. PQRS is a rectangle with sides 3cm and 4cm, if its diagonal cross at O, calculate the length of PO. a) 3.5cm b) 5.0cm c) 2.5cm d) 4.0cm
4. If the angles of a pentagon could be x , $2x$, $4x$ and $5x$ respectively, what would be the value of x ? a) 60° b) 90° c) 15° d) 30°

THEORY

1. From the top of a building 50m high, the angle of depression of a car is 55° , find the distance of the car from the foot of the building.
2. Find the height of the flagpole in the diagram below to the nearest metre.



WEEK NINE

DATE-----

TOPIC: BEARING

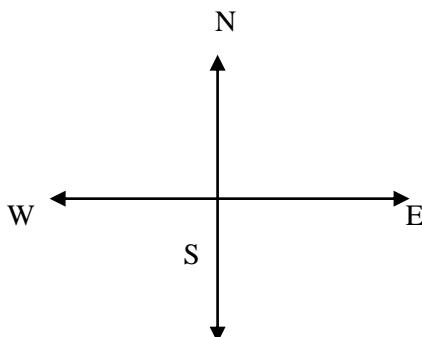
BEARING

- CONTENT:**
- i) Compass bearing
 - ii) Three figure bearing
 - iii) Finding the bearing of a point from another.

Compass Bearing

A bearing gives the direction between two points in terms of an angle in degrees. The two types of bearings are compass bearings and three figure bearings.

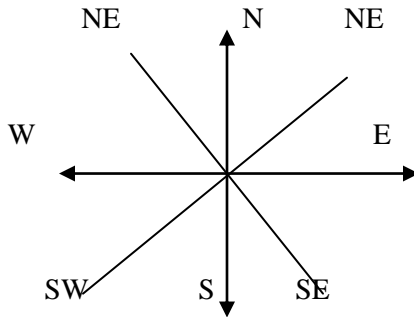
The four major compass directions are North (N), South (S), East (E), and West (W).



Name: _____

Class: _____

In compass bearings, the angles are measured from North or South depending on which one is nearer



Apart from the four main points or directions, there are also four main secondary directions i.e NE (North East), SE (South East), SW (South West), NW (North West). The angle between each point is 45° .

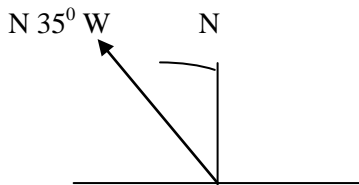
Worked Examples

Draw a sketch to show each of these bearings marking the angles clearly.

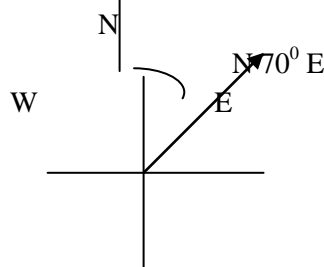
- a) $N 35^\circ W$ b) $N 70^\circ E$ c) $S 58^\circ W$.

Solution

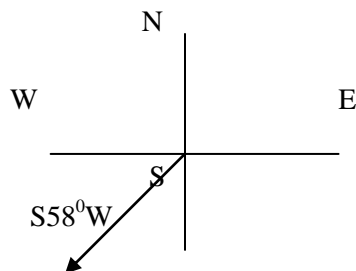
- a) $N 35^\circ W$ means from N, measures 35° towards the W or $35^\circ W$ of N.



- b) $N 70^\circ E$ means 70° towards E.

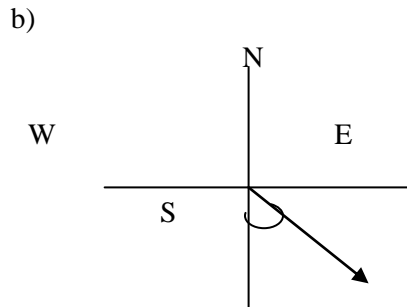
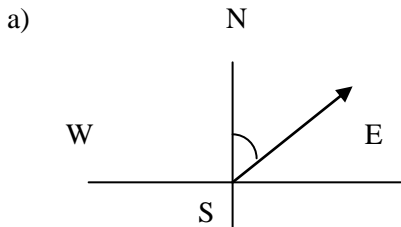


- c) $S 58^\circ W$ means from S measures 58° towards W.



Evaluation:

Find the compass direction of point A from point O in these diagrams.

**REFERENCE**

NGM Bk 2 Chapter 23, Page 185 – 187.

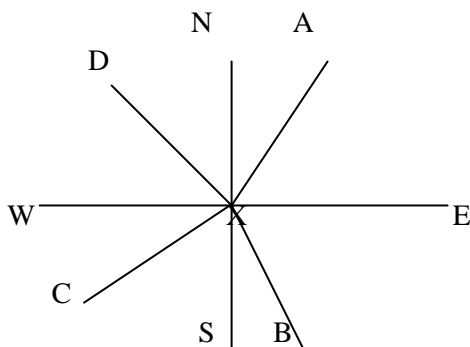
Essential Mathematics for JSS Bk 2, Chapter 24, Pg 246 - 247.

THREE FIGURE BEARINGS

Three figure bearings are given as the number of degrees from North, measured in a clockwise direction. Any direction can be given as a three figure bearing. Three digits are always given but angles less than 100° need extra zero to be written in front of the digits e.g. 000° , 036° , 070° up to 099° .

Worked Examples

Find the three figure bearings of A, B, C, and D from X.

**Solution**

a) The arrow N shows the direction N, $NXA = 63^{\circ}$. The bearing of A from X is 063° .

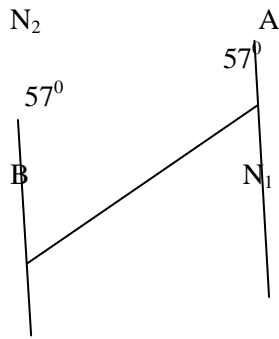
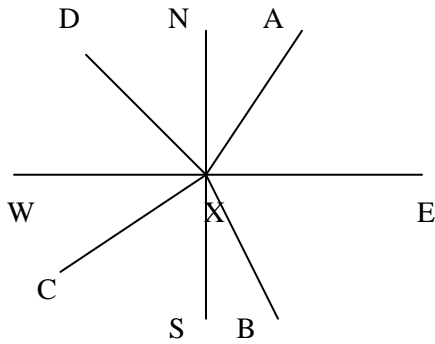
b) $NXB = 180 - 35 = 145^{\circ}$. The bearing of B from X is 145° .

c) NXC clockwise = $180 + 75 = 255^{\circ}$. The bearing of C from X is 255° .

d) NXD clockwise = $360 - 52 = 308^{\circ}$. The bearing of D from X is 308° .

Evaluation

In the figure below, find the bearings of A, B, C and D from X.



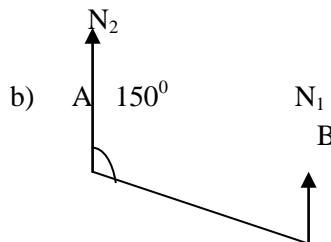
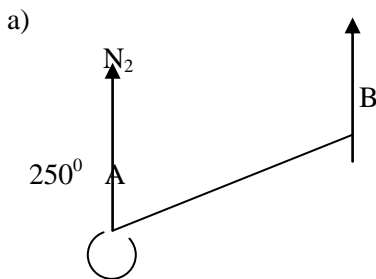
By constructing line N_2A

$\angle N_2BA$ is 57° , similarly, $N_1AB = 57^\circ$ (alternate angles are equal). From point A, starting from the North, $180 + 57 = 237^\circ$.

- a) The bearing of B from A is 237°
- b) The bearing of A from B is 057° .

Evaluation:

1. The bearing of X from Y is 319° . Calculate the bearing of Y from X.
2. In each diagram, calculate (i) the bearing of B from A and (ii) the bearing of A from B



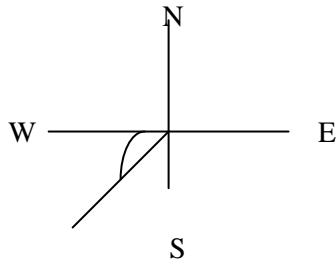
READING ASSIGNMENT

NGM Bk 2 Chapter 23, Page 189 – 190

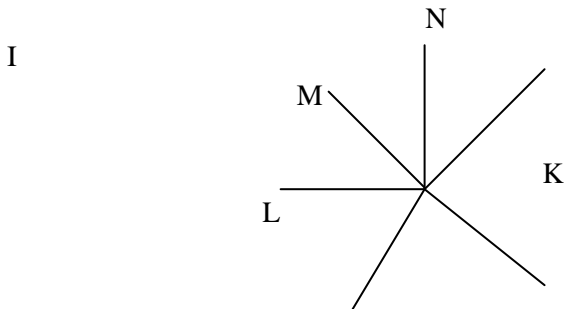
Essential Mathematics for JSS Bk 2, Chapter 24, Pg 248 - 250.

WEEKEND ASSIGNMENT

1. If the angle of elevation of P from Q is 38° , then the angle of depression of Q from P is _____
 a) 19° b) 52° c) 38° d) 128°
2. The compass direction of the diagram below is



- a) $S20^\circ W$ b) $S60^\circ W$ c) $S70^\circ$ d) $S20^\circ E$
3. Use the diagram below to answer questions 3 to 5.



The bearing of J from X is _____

- a) 61° b) 103° c) 42° d) 188°
4. The bearing of L from X is _____
 a) 260° b) 160° c) 100° d) 76°
5. The bearing of M from X is _____
 a) 226° b) 300° c) 336° d) 24°

THEORY

1. Bala stood on one side of a field and finds that the bearing of an orange trees is $N75^\circ W$.
 Shortly after, Asiru moved to the foot of the orange tree to spot Bala. Calculate bala's bearing from asiru. X
2. The bearing of P from Q is 195° , what would be the bearing of Q from P?

WEEK TEN

DATE-----

SCALE DRAWING**Using Scales**

A scale is a ratio or proportion that shows the relationship between a length or a drawing and the corresponding length on the actual object.

Thus,

$$\text{Scale} = \frac{\text{Any length on scale drawing}}{\text{Corresponding length on actual object}}$$

Worked Examples

1. The scale drawing of the length of an advertisement billboard measures 5cm. What is the actual length of the billboard if the scale is 1cm represents 2m?

Solution

Name: _____

Class: _____

1cm represents 2m

5cm represents $5 \times 2\text{m} = 10\text{m}$

The actual length of the billboard = 10m.

2. An airport runway measuring 6000m is drawn to a scale of 1cm represents 500m. Find its length on the drawing.

Solution

500m is represented by 1cm

1m is represented by $1/500\text{cm}$

6000m is represented by $6000 \times 1/500 = 12\text{cm}$

Length on drawing = 12cm

Evaluation:

1. Copy and complete the table below in finding the length on drawing giving a suitable scale.

Actual length	Scale	Length on drawing
20m	1cm to 5m	
450m	1cm to 100m	
65m	1cm to 5m	

2. Copy and complete the table by finding the actual length.

Length on drawing	Scale	Actual length
1 cm	1cm to 5m	
8.2cm	1cm to 100m	
12.6cm	2cm to 1m	

Scale Drawing

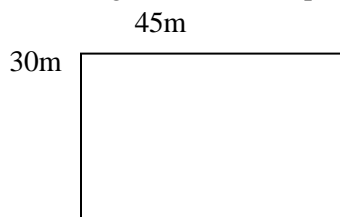
Scale drawing is very important to engineers, architects, surveyors and navigators. For an accurate scale drawing, mathematical instruments are needed such as pencils, a ruler and a set-square. Also, the dimensions of the actual objects are written on the drawing.

Worked Examples

1. A rectangular field measures 45m by 30m. Draw a plan of the field. Use measurement to find the distance between opposite corners of the field.

Solution

Firstly, make a rough sketch of the plan



Secondly, choose a suitable scale

Using 1cm represent 5m will give a 9cm by 6cm rectangle.

The distance between the opposite corners of the field is represented by the dotted line. Length of the dotted line = 10.75

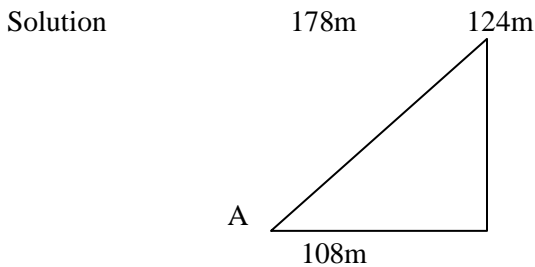
Actual distance = $10.75 \times 5 = 53.75\text{m} = 54\text{m}$ (to the nearest metre)

Name: _____

Class: _____

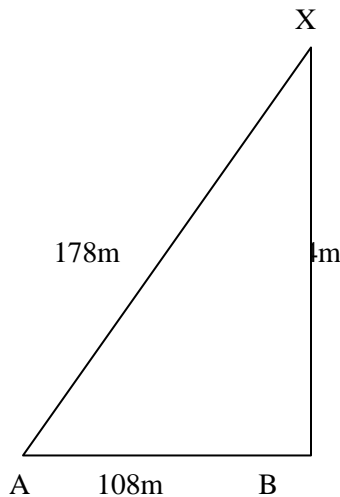
Example 2:

Points A and B are 178m and 124m from X respectively. The distance between A and B is 108m. Make a scale drawing of the path and find the angle between the paths and X. X



Using a suitable scale of 1cm to 20m, the sides of the triangle in scale drawing will be as follows:

$AX = 178/20 = 8.9\text{cm}$, $BX = 124/20 = 6.2\text{cm}$, $AB = 108/20 = 5.4\text{cm}$.



Using a protractor, $\angle AXB = 37^\circ$ (to the nearest degree). The angle between the paths is 37° .

Evaluation:

- 1) Find the distance between the opposite corners of a rectangular room which is 12m by 9m. Use a scale of 1cm to 3m.
- 2) A triangular plot ABC is such that $AB = 120\text{m}$, $BC = 80\text{m}$ and $CA = 60\text{m}$. P is the middle point of AB. Find the length of PC. Use a scale of 1cm to 10m.

Application of Scale Drawing on Related Problems

Worked Examples

- 1) The scale on a map is 1 : 50,000.
 - a) Two villages A and B on the map are 5.5cm apart, find the actual distance in km between A and B.
 - b) If town C is 4km from the village A, what is the distance of C from A on the map?

Solution

Note: Map scale = actual distance/distance on the map.

\therefore Distance on map = Actual distance / Map Scale

- a. 1cm represents 50,000cm
5.5cm represent $50,000 \times 5.5 = 275,000\text{cm}$
To correct to km = $275,000/100,000 = 2.75\text{km}$.
- b. 1km = 100,000cm
 $4\text{km} = 100,000 \times 4 = 400,000\text{cm}$.
Since 50,000cm represents 1cm
400,000cm is represented by $400,000/50,000 = 8\text{cm}$
or Distance on the map = Actual distance / Map Scale

Name: _____

Class: _____

$$= 400,000 / 50,000 = 8\text{cm.}$$

Example 2:

Two cities are 70km apart. The distance between them is 20cm on the map. What is the scale of the map?

Solution

$$1\text{km} = 100,000\text{cm}$$

$$\therefore 70\text{km} = 100,000 \times 70 = 7,000,000\text{cm}$$

Map scale = actual distance/distance on map.

$$= 7,000,000/20 = 350,000$$

The scale of the map = 1 : 350,000.

Evaluation: Class Work

- The scale on the map is 1 : 25,000.
 - Find the distance in km between two islands represented by a distance of 20cm on the map.
 - Find the distance between two towns on the map that are 10km apart..
- The scale of a map is 1 : 20,000. Find the actual distance in km represented by the map by
 - 5cm
 - 10cm

READING ASSIGNMENT:

NGM Bk 2 Chapter 16, Pages 133 – 134.

Essential Mathematics for JSS Bk 2, Chapter 17, Pg 166 – 169

WEEKEND ASSIGNMENT

- A quadrilateral has angles of 128° , 91° , a° and $2a^\circ$. Find the value of a° .
 - 67°
 - 47°
 - 57°
 - 107°
- The sum of the angles of a polygon is 1620° , calculate the number of sides that the polygon has.
 - 11
 - 21
 - 16
 - 13
- In fig. 1, PQRT and TQRS are parallelogram
QR = 3cm and TQ = 4cm, what is PS?
- Which of the following are Pythagoras triple? I (3, 4, 5) II (5, 12, 13) III (8, 13, 17).
 - III only
 - I and II only
 - II only
 - II and III only.
- The diagonals of a rhombus measures 8cm y 6cm, what is the length of a side of the rhombus?
 - 8cm
 - 7cm
 - 10cm
 - 5cm

THEORY

1.The scale on the map is 2cm : 30,000km

Find the distance in km between two islands represented by a distance of 20cm on the

2. A map of Nigeria shows scale of 1cm representing 75km. how far is it from Ibadan to kano, if the route distance measures 30.5cm on the map