MR OSHO TUTORS
Researcher || Educator || Career coach
Content builder || Mathematician
08135056422
Mroshotutors@gmail.com

SECOND TERM E-LEARNING NOTE

SUBJECT: MATHEMATICS **CLASS: SS 1**

SCHEME OF WORK

WEEK	**TOPIC**
1.	Quadratic Equation by (a) Factorization (b) Completing the square method
2.	General Form of Quadratic Equation leading to Formular Method $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$from $ax^2 + bx + c = 0$
3.	Solutions of Quadratic Equation by Graphical Methods: (a) Reading the Roots from the Graph (b) Determination of the Minimum and Maximum Values (c) Line of Symmetry.
4.	Idea of Sets: (a) Universal Sets, Finite and Infinite Sets, Empty Set, Subset (b) Idea of Notation for Union and Intersection of Sets
5.	Complements of Sets: (a) Disjoints of Null. (b) Venn Diagramand its Use in Solving Problems Involving two and three Sets Relation to Real Life Situations.
6.	Review of the First Half Term's Work and Periodic Test
7.	Trigonometric Ratios (a) Sine, Cosine, Targentof Acute Angles (b) Use of Tables of Trigonometric Ratios (c) Determination of Length of Chord (d) Using Trigonometric Ratios (e) Graph of Sine and Cosine for Angles $0^o = x$
8.	(a) Application of Sine, Cosine and Tangent, Simple Problems with Respect to Right Angle Triangles. (b) Angles of Elevation and Depression (c) Bearing and Distances of Places Strictly Application of Trigonometric Ratio.
9.	(a) Introduction of Circle and its Properties (b) Calculation of Length of Arc and Perimeter of a Sector (c) Area of Sectors and Segments. Area of triangles
10.	Logic (a) Simple True and False Statements (b) Negative and Contra Positive of Simple Statement. (c) Antecedents, Consequence and Conditional Statement (implication)

REFERENCE BOOK

- New General Mathematics SSS 1 M.F. Macrae et al
- WABP Essential Mathematics For Senior Secondary Schools 1 A.J.S Oluwasanmi

WEEK ONE

Topic: **Quadratic equation by (a) Factorization (b) Completing the square method**

Quadratic Equations
A quadratic equation contains an equal sign and an unknown raised to the power 2. For example:
$2x^2 - 5x - 3 = 0$
$n^2 + 50 = 27n$
$0 = (4a - 9)(2a + 1)$
$49 = k^2$

Are all quadratic equations.
Discussion: can you see why
$0 = (4a - 9)(2a + 1)$ is a quadratic equation?
One of the main objectives of the chapter is to find ways of solving quadratic equations,
i.e. finding the value(s) of the unknown that make the equation true.

Solving Quadratic Equations
One way of solving quadratic equation is to apply the following argument to a quadratic expression that has been factorized.
If the product of two numbers is 0, then one of the numbers (or possibly both of them) must be 0. For example,
$3 \times 0 = 0$, $0 \times 5 = 0$ and $0 \times 0 = 0$
In general, if $a \times b = 0$
Then either $a = 0$
Or $b = 0$
Or both a and b are 0

Example 1
Solve the equation $(x - 2)(x + 7) = 0$.
If $(x - 2)(x + 7) = 0$
Then either $x - 2 = 0$ or $x + 7 = 0$
$x = 2$ or -7

Example 2
Solve the equation $d(d - 4)(d + 6^2) = 0$.
$(3a + 2)(2a - 7) = 0$, then any one of the four factors of the LHS may be 0,
i.e $d = 0$ or $d - 4 = 0$ or $d + 6 = 0$ twice.
$\Rightarrow d = 0, 4$ or -6 twice.

EVALUATION
Solve the following equations.
1. $3d^2(d - 7) = 0$
2. $(6 - n)(4 + n) = 0$
3. $A(2 - a)^2(1 + a) = 0$

Solving quadratic equations using factorization method
The LHS of the quadratic equation $m^2 - 5m - 14 = 0$ factorises to give $(m + 2)(m - 7) = 0$.

Example 1

Solve the equation $4y^2 + 5y – 21 = 0$

$4y^2 + 5y – 21 = 0$

$\Longrightarrow$ $(y + 3)(4y – 7) = 0$

$\Longrightarrow$either $y + 3 = 0$ or $4y – 7 = 0$

$y = - 3$ or $4y = 7$

$y = - 3$ or $y = 7/4$

$y = -3$ or $1\frac{3}{4}$

check: by substitution:

if $y = -3$

$4y^2 + 5y – 21 = 36 – 15 – 21 = 0$

If $y = 1\frac{3}{4}$,

$4y^2 + 5y – 21 = 4 \text{ x } 7/4 \text{ x } 7/4 + 5 \text{ x } 7/4 – 21$

$= \frac{49}{4} + \frac{35}{4} – 21 = 0$

Example 2

Solve the equation $m^2 = 16$

Rearrange the equation.

If $m^2 = 16$

Then $m^2 – 16 = 0$

Factorise (difference of two squares)

$(m - 4)(m + 4) = 0$

Either $m – 4 = 0$ or $m + 4 = 0$

$m = +4$ or $m = -4$

$m = \pm 4$

EVALUATION

Solve the following quadratic equations:

1. $h^2 – 15h + 54 = 0$
2. $12y^2 + y – 35 = 0$
3. $4a^2 – 15a = 4$
4. $v^2 + 2v – 35 = 0$

GENERAL EVALUATION

Solve the following equations:

1. $y^2(3 + y) = 0$
2. $x^2(x + 5)(x - 5) = 0$
3. $(v - 7)(v - 5)(v - 3) = 0$
4. $9f^2 + 12f + 4 = 0$

WEEKEND ASSIGNMENT

Solve the following equations. Check the results by substitution.

1. $(4b - 12)(b - 5) = 0$ A. ½, 4 B. 3, 5 C. 4, 6 D.5, 3
2. $(11 – 4x)^2 = 0$ A.$\frac{11}{3}$, 3 B.$2\frac{3}{4}$, 3 C. $2\frac{3}{4}$ twice D. $2\frac{4}{3}$ twice
3. $(d – 5)(3d – 2) = 0$ A. 5,$\frac{2}{3}$ B. 4, 5 C. 5, 9 D. $\frac{2}{3}$, 5

Solve the following quadratic equations

4. $u^2 - 8u - 9 = 0$ A. – 9, 1 B. -1, 9 C. 1, 8 D. 9 , -1
5. $c^2 = 25$ A. 5 B. -5 C.+5 D.± 5

THEORY

Solve the equation

1. $2x^2 = 3x + 5$
2. $a^2 - 3a = 0$
3. $p^2 + 7p + 12 = 0$

WEEK TWO

TOPIC:General form of quadratic equation leading to Formular method

CONTENT

- Derivative of the Roots of the General Formof Quadratic Equation.
- Using the FormularMethods to solve Quadratic Equations
- Sum and Product of quadratic roots.

Derivative of the Roots of the General Form of Quadratic Equation

The general form of a quadratic equation is $ax^2 + bx + C = 0$. The roots of the general equation are found by completing the square.

$ax^2 + bx + C = 0$

Divide through by the coefficient of x^2.

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{C}{a} = 0$$

$$x^2 + \frac{bx}{a} + \frac{C}{a} = 0$$

$$x^2 + \frac{b}{a}x = 0 - \frac{C}{a}$$

$$x^2 + \frac{bx}{a} = -\frac{C}{a}$$

The square of half of the coefficient of x is

$$\left(\frac{1}{2} \times \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2$$

Add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{C}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left[x+\frac{b}{2a}\right]^2 = -\frac{C}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{-4ac+b^2}{4a^2}$$

i.e $$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

Take square roots of both sides of the equation :

$$\sqrt{\left[x+\frac{b}{2a}\right]^2} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

i.e $$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = -\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

Hence

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

EVALUATION

Suppose thegeneral quadratic equation is $Dy^2 + Ey + F = 0$

Using the method of completing the square, derive the roots of this equation

Using the FormularMethods to Solve Quadratic Equations

Examples

Use the formula method to solve the following equations. Give the roots correct to 2 decimal places:

i. $3x^2 - 5x - 3 = 0$
ii. $6x^2 + 13x + 6 = 0$
iii. $3x^2 - 12x + 10 = 0$

Solution

1. $3x^2 - 5x - 3 = 0$

Comparing $3x^2 - 5x - 3 = 0$

With $ax^2 + bx + C = 0$

$a = 3, b = -5, C = -3$

Since

$$X = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-5)\pm\sqrt{(-5)^2 - 4 \times 3 \times -3}}{2 \times 3}$$

$$x = \frac{+5\pm\sqrt{25+36}}{6}$$

$$x = \frac{+5\pm\sqrt{61}}{6}$$

$x = \frac{+5 + 7.810}{6} = \frac{+12.810}{6}$

or

$x = \frac{+5 - 7.810}{6} = \frac{-2.810}{6}$

$x = \frac{12.810}{6}$ or $x = -\frac{2.810}{6}$

$x = \frac{12.810}{2}$ or $x = -\frac{2.810}{6}$

i.e. $x = 2.135$ or $x = -0.468$

$x = 2.14$ or $x = -0.47$

to 2 decimal places

(2) $6x^2 + 13x + 6 = 0$

comparing $6x^2 + 13x + 6 = 0$

with $ax^2 + bx + c = 0$

$a = 6$, $b = 13$, $c = 6$

Since

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-13 \pm \sqrt{(13)^2 - 4 \times 6 \times 6}}{2 \times 6}$

$x = \frac{-13 \pm \sqrt{169 - 144}}{12}$

$x = \frac{-13 \pm \sqrt{25}}{12}$

$x = = \frac{-13 \pm 5}{12}$

$x = \frac{-13 + 5}{12}$ or $x = \frac{-13 - 5}{12}$

$x = -\frac{8}{12}$ or $x = -\frac{18}{12}$

$x = -\frac{2}{3}$ or $x = \frac{-3}{2}$

$x = -0.666$ or $x = -1.50$

i.e $x = 0.67$ or $x = -150$ to 2 decimal places .

(3) $3x^2 - 12x + 10 = 0$

comparing $3x^2 - 12x + 10 = 0$ with $ax^2 + bx + c = 0$, then

$a = 3$, $b = -12$, $c = 10$.

Since

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

then

$x = \frac{-(-12) \pm \sqrt{(-12)2 - 4 \times 3 \times 10}}{2 \times 3}$

$x = \frac{+12 \pm \sqrt{144 - 120}}{}$

6

$x = \frac{+12 \pm \sqrt{24}}{6}$

$x = \frac{12 \pm 4.899}{6}$

$x = \frac{+12 + 4.899}{6} = \frac{16.899}{6}$

or $x = \frac{+12 - 4.899}{6} = \frac{7.101}{6}$

i.e $x = \frac{16.899}{6}$ or $x = \frac{7.101}{6}$

$x = 2.8165$ or $x = 1.1835$

i.e . $x = 2.82$ or $x = 1.18$ to 2 decimal places.

EVALUATION

Use the formula method to solve the following quadratic equations .

1. $t^2 - 8t + 2 = 0$
2. $t^2 + 3t + 1 = 0$

i. **Sum and Product of quadratic roots.**

We can find the sum and product of the roots directly from the coefficient in the equation
It is usual to call the roots of the equation α and β If the equation

$ax^2 + bx + c = 0$ ……………. I

has the roots α and β then it is equivalent to the equation

$(x - \alpha)(x - \beta) = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ ………… 2

Divide equation (1)by the coefficient of x^2

$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$ ………… 3

Comparing equations (2) and (3)

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

then

$\alpha + \beta = \frac{-b}{a}$

and $\alpha\beta = \frac{c}{a}$

For any quadratic equation, $ax^2 + bx + c = 0$ with roots α and β

$\alpha + \beta = \frac{-b}{a}$

$\alpha\beta = \frac{C}{a}$

Examples

1. If the roots of $3x^2 - 4x - 1 = 0$ are α and β, find $\alpha + \beta$ and $\alpha\beta$

2. If α and βare the roots of the equation
$3x^2 - 4x - 1 = 0$, find the value of

(a) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(b) $\alpha - \beta$

Solutions

1a. Since $\alpha + \beta = \frac{-b}{a}$

Comparing the given equation $3x^2 - 4x - 1 = 0$ with the general form $ax^2 + bx + c = 0$

$a = 3, b = -4, c = 1.$

Then

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{3}$

$= +\frac{4}{3} = +1\,{}^1/_3$

$\alpha\beta = \frac{c}{a} = \frac{-1}{3} = \frac{-1}{3}$

2. (a) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

Here, comparing the given equation, with the general equation,

$a = 3, b = -4, c = -1$

from the solution of example 1 (since the given equation are the same),

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{3} = +\frac{4}{3}$

$\alpha\beta = \frac{c}{a} = -\frac{1}{3}$

then

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(4/3)^2 - 2(-1/3)}{-1/3}$

$= \frac{\frac{16}{9} + \frac{2}{3}}{-\frac{1}{3}}$

$= \frac{16 + 6}{9} \div \frac{-1}{3}$

$\frac{22}{9} \times \frac{-3}{1}$

$= \frac{-22}{3}$

or $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{22}{3} = -7\ 1/3$

(b) Since

$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

but

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\therefore (\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\therefore (\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$(\alpha - \beta) = \sqrt{(4/3)^2 - 4(-^1/3)}$

$= \sqrt{16/9 + ^4/3}$

$= \sqrt{\frac{16+12}{9}}$

$= \sqrt{\frac{28}{9}} \quad = \frac{\sqrt{28}}{3}$

$\therefore \alpha - \beta = \frac{\sqrt{28}}{3}$

EVALUATION

If α and β are the roots of the equation $2x^2 - 11x + 5 = 0$, find the value of

a. $\alpha - \beta$

b. $\frac{1}{\alpha+1} + \frac{1}{\beta+1}$

GENERAL EVALUATION

Solve the following quadratic equations:

1. $63z = 49 + 18z^2$
2. $8s^2 + 14s = 15$

Solve the following using formula method:

3. $12y^2 + y - 35 = 0$
4. $h^2 - 15h + 54 = 0$

READING ASSIGNMENT

New General Mathematics SS Bk2 pages 41-42 ,Ex 3e Nos 19 and 20 page 42.

WEEKEND ASSIGNMENT

If α and β are the roots of the equation $2x^2 - 7x - 3 = 0$ find the value of:

1. $\alpha + \beta$ (a) $^2/3$ (b) $^7/2$ (c) $^2/5$ (d) $^5/3$
2. $\alpha\beta$ (a) $^{-3}/2$ (b) $^2/3$ (c) $^3/2$ (d) $-^2/3$
3. $\alpha\beta^2 + \alpha^2\beta$ (a) $^{21}/4$ (b) $^4/21$ (c) $^{-4}/21$ (d) $^{-21}/4$

Solve the following equation using the formula method.

4. $6p^2 - 2p - 7 = 0$
5. $3 = 8q - 2q^{2.}$

THEORY

1. Solve the equation $2x^2 + 6x + 1 = 0$ using the formula method
2. If α and β are the roots of the equation $3x^2 - 9x + 2 = 0$, find the values of

i. $\alpha \beta^2 + \alpha^2\beta$
ii. $\alpha^2 - \alpha\beta + \beta^2$

WEEK THREE
Topic:Solution of quadratic equation by graphical method.
CONTENT

- Reading the roots from the graph
- Determination of the minimum and maximum values
- Line of symmetry.

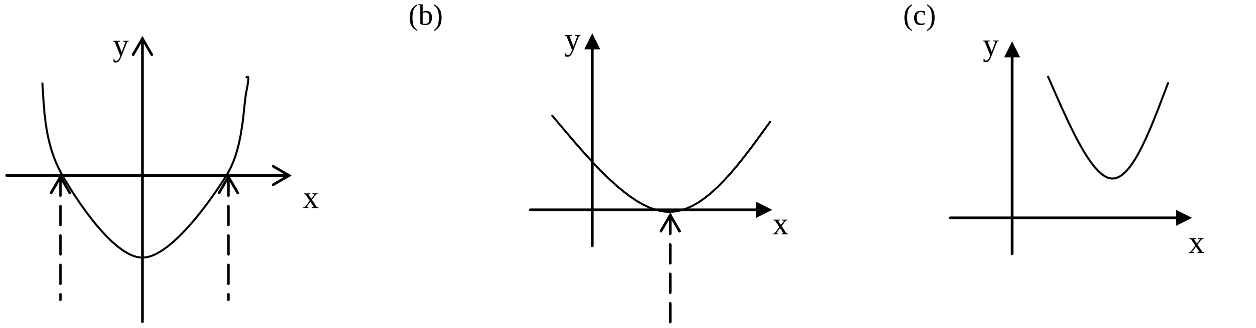
The following steps should be taken when using graphical method to solve quadratic equation :

i. Use the given range of values of the independent variable (usually x) to determine the corresponding values of the dependent variable (usually y) by the quadratic equation or relation given. If the range of values of the independent variable is not given, choose a suitable one.
ii. From the results obtained in step (i), prepare a table of values for the given quadratic expression.
iii. Choose a suitable scale to draw your graph.
iv. Draw the axes and plot the points.
v. Use a broom or flexible curve to join the points to form a smooth curve.

Notes

1. The roots of the equation are the points where the curve cuts the x – axis because along the x- axis y = 0
2. The curve can be an inverted n – shaped parabola or it can be a v-shaped parabola. It is n-shaped parabola when the coefficient of x^2 is negative and it is V- shaped parabola when the coefficient of x^2 is positive. Maximum value of y occurs at the peak or highest point of the n-shaped parabola while minimum value of y occurs at the lowest point of V-shaped parabola.
3. The curve of a quadratic equation is usually in one of three positions with respect to the x – axis.

(a) (b) (c)



In fig(a), the curve crosses the x-axis at two clear points. These two points give the roots of the quadratic equation.In fig (b), the two points are coincident, i.e their points are so close together that the curve touches the x axis at one point. This corresponds to an equation which has one repeated root.
In fig (c), the curve does not cut the x-axis. The roots of an equation which gives a curve in such a position are said to be imaginary.
4. The line of symmetry is the line which divides the curve of the quadratic equation into two equal parts.

Examples

1a. Draw the graph of $y = 11 + 8x - 2x^2$ from $x = -2$ to $x = +6$.

b. Hence find the approximate roots of the equation $2x^2 - 8x - 11 = 0$

c. From the graph, find the maximum value of y.

2a. Given that $y = 4x^2 - 12x + 9$, copy and complete the table below

X	-1	0	1	2	3	4
$4x^2$	4			16		64
-12x	12			-24		-48
+9	9			9		9
Y	25			1	3	25

b. Hence draw a graph and find the roots of the equation $4x^2 - 12x + 9 = 0$

c. From the graph, what is the minimum value of y ?

d. From the graph, what is the line of symmetry of the curve?

Solutions

$Y = 11 + 8x - 2x^2$
from $x = -2$ to $x = +6$
When $x = -2$
$Y = 11 + 8(-2) - 2(-2)^2$
$Y = 11 - 16 - 2(+4)$
$Y = 11 - 16 - 8$
$Y = -5 - 8 = -13.$

When $x = -1$
$Y = 11 + 8(-1) - 2(-1)^2$
$Y = 11 - 8 - 2(+1)$
$Y = 11 - 8 - 2$
$Y = 3 - 2 = 1.$

When $x = 0$
$Y = 11 + 8(0) - 2(0)^2$
$Y = 11 + 0 - 2 \times 0$
$Y = 11 + 0 - 0$
$Y = 11$

When $x = 1$
$Y = 11 + 8(1) - 2(1)^2$
$Y = 11 + 8 - 2 \times 1$
$Y = 19 - 2 = 17$
When $x = 2$
$Y = 11 + 8(2) - 2(2)^2$
$= 11 + 16 - 2 \times 4$
$= 27 - 8 = 19$

when $x = 3$
$y = 11 + 8(3) - 2(3)^2$
$= 11 + 24 - 2 \times 9$
$= 35 - 18 = 17$

when x = 4
$y = 11 + 8(4) - 2(4)^2$
$= 11 + 32 - 2 \times 16$
$= 43 - 32 = 11$

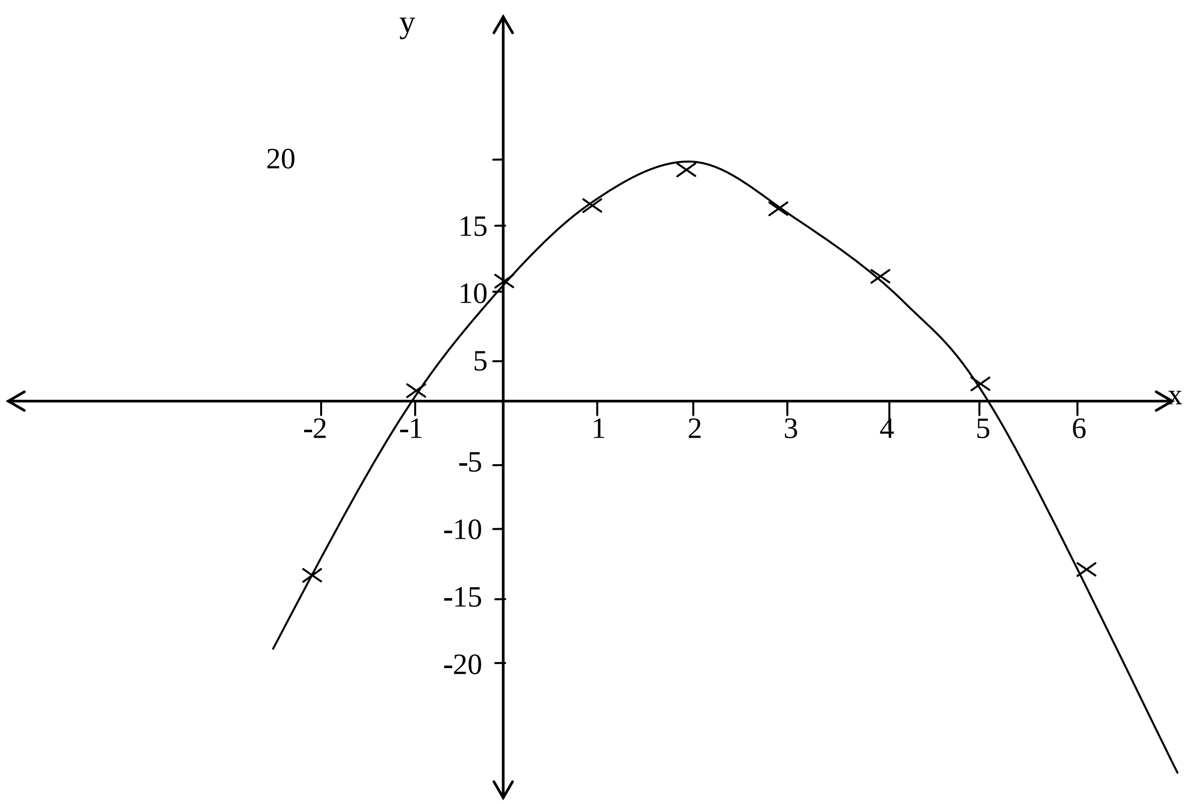
when x = 5
$y = 11 + 8(5) - 2(5)^2$
$= 11 + 40 - 2 \times 25$
$= 51 - 50 = 1$

when x = 6
$y = 11 + 8(6) - 2(6)$
$= 11 + 48 - 2 \times 36$
$= 59 - 72$
$= -13$

The table of values is given below :

X	-2	-1	0	1	2	3	4	5	6
Y	-13	1	11	17	19	17	11	1	-13

Scale
On x axis, let 2cm = 1 unit; on y axis, let 1cm = 5 units



b. From the graph, the approximate roots of the equation are the points where the curve cuts the x axis,this is so because

$y = 11 + 8x - 2x^2$

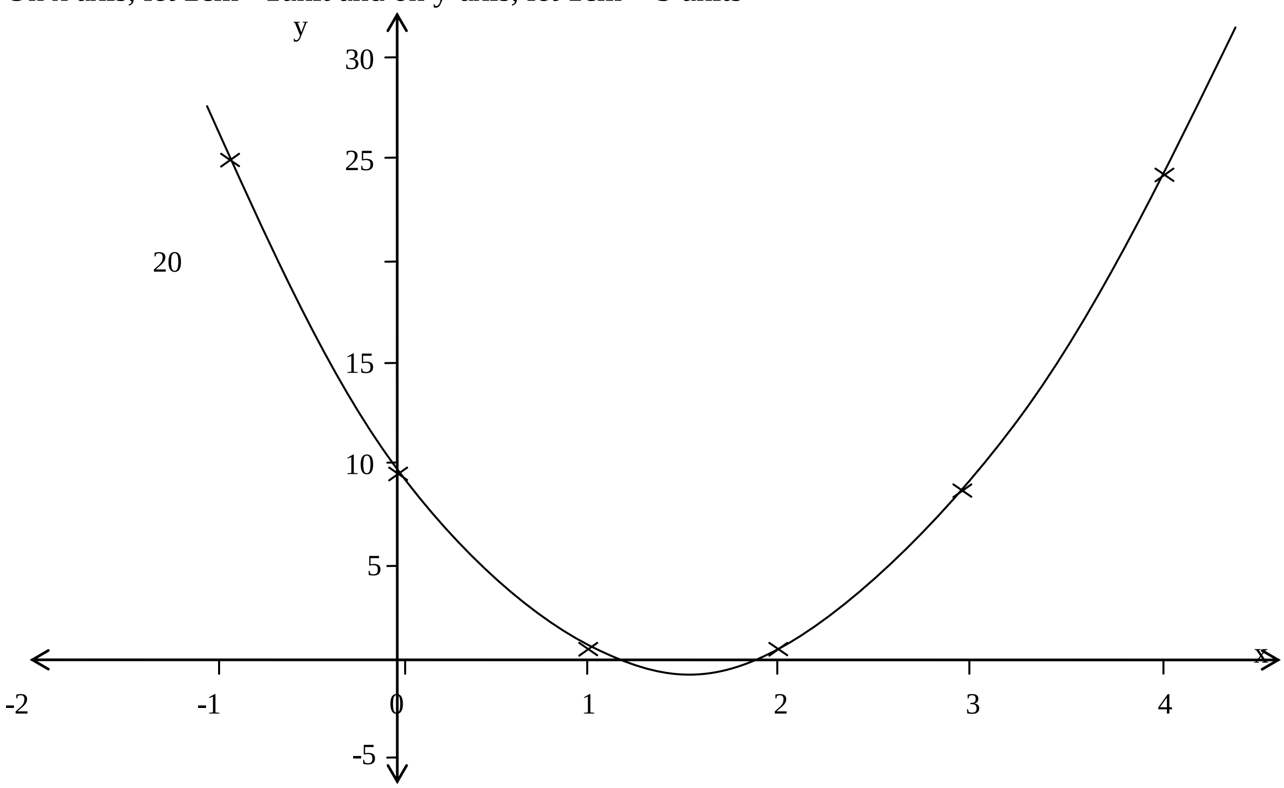
$-1 \times y = -1 \times (11) + 8x(-1) - 2x^2(-1)$

$-y = -11 - 8x + 2x^2$
$-y = 2x^2 - 8x - 11 = 0$
$-1x - y = 0 \times -1$
i.e $y = 0$
Thus, from the graph, the roots of the equation $2x^2 - 8x - 11 = 0$ are $x = -1.1$or $x = 5.1$
c. The maximum value of $y = 19$.

2 a. The completed table is given as follows

X	-1	0	1	2	3	4
$4x^2$	4	0	4	16	36	64
-12x	12	0	-12	-24	-36	-48
+9	9	9	9	9	9	9
Y	25	9	1	1	9	25

Scale
On x axis, let 2cm =1unit and on y-axis, let 1cm = 5 units



From the graph, the roots of the equation is the points where the curve touches the x axis i.e $x = 1.5$ twice

c. Fromthe graph, the minimum value of $y = 0$
d. Fromthe graph, the line of symmetry of the curve is line $x = 1.5$

EVALUATION
a. Usinga suitable scale, draw the graph of $y = x^2 - 2x$ from $x = -2$ to $x = + 4$
b. From the graph, find the approximate roots of the equation
$x^2 - 2x = 0$

c. What is the minimum value of y ?
d. Find the values of x when $y = 7$.

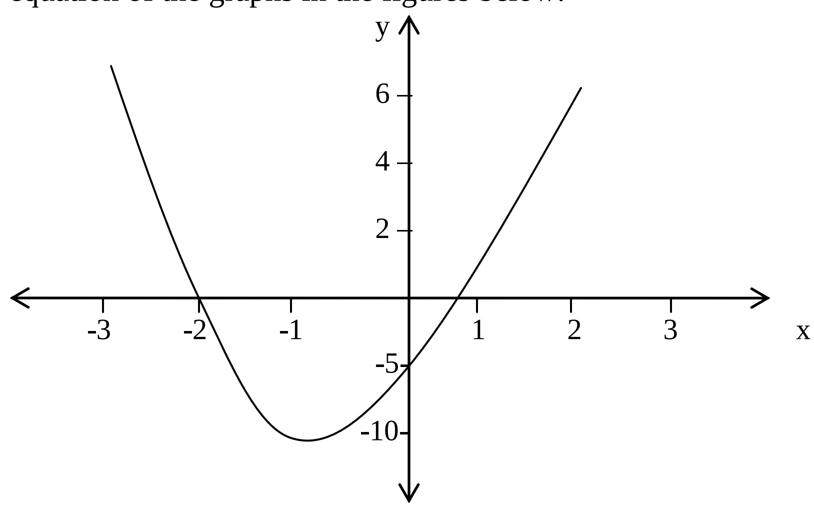
Finding an equation from a given graph

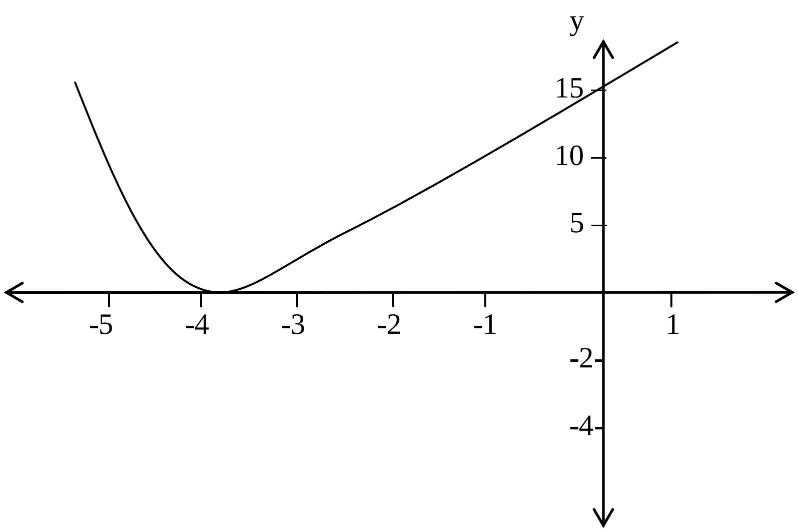
In general, if a graph (curve) cuts the x axis, at points a and b, the required equation is obtained from the expression $(x - a)(x - b) = 0$

Examples

Find the equation of the graphs in the figures below:

Fig. 1





Solutions

1. First in figure 1 when $y = 0$, $x = -2$ and $x = ½$

Hence

$$\left(x - (-2) \right) \quad \left(x - ½ \right) = 0$$

$$x + 2 \left(x - ½ \right) \left(= 0 \right)$$

$\left[x\left(x-\tfrac{1}{2}\right)\right] \left[+2\left(x-\tfrac{1}{2}\right)\right] = 0$

$x^2 - {}^1/_2x + 2x - 1 = 0$

$x^2 + 1\tfrac{1}{2}x - 1 = 0$ ………… (1)

Second:

At the intercept in y axis

$y = -2$ when $x = 0$

However, the constant term in equation (1) is – 1

Then, multiply both sides of the equation (1) by 2

i.e $2x^2 + 3x - 2 = 0$ ……………2

Equation (2) satisfies

$\left(x - 1/2\right)\left(x - (-2)\right) = 0$

and the requirement that the constant term should be -2

:. The equation of the curve is $y = 2x^2 + 3x - 2 = 0$

2. First in fig (2), the curve just touches the x axis at the point $x = -4$. Since a quadraticic equation has two roots, this implies that the root are repeated when $y = 0$

i.e when $y = 0$, $x = -4$ (twice)

So the equation must satisfy

$\left(x - (-4)\right) \quad \left(x - (-4)\right) = 0$

$\left(x + 4\right) \quad \left(x + 4\right) = 0$

$x\left(x + 4\right) + 4\left(x + 4\right) = 0$

$x^2 + 4x + 4x + 16 = 0$

$x^2 + 8x + 16 = 0$ ………..1

Second:

At the intercept on y- axis

$y = 15$ when $x = 0$

However, the constant term in equation (1) is + 16. Then multiply both sides of the equations (i) by – 1.

i.e $-x^2 - 8x - 16 = 0$ ………..2

Thus equation 2 satisfies

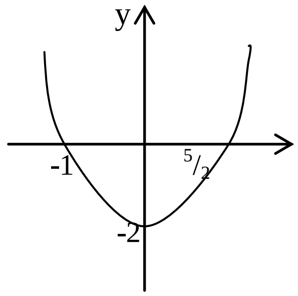
$(x + 4)(x + 4) = 0$ and the requirement that the constant term should be - 16.

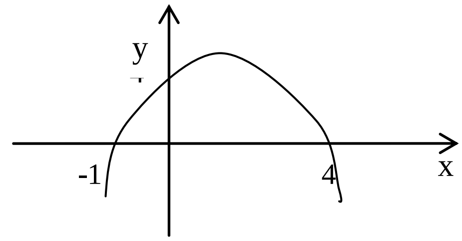
:. The equation of the curve is

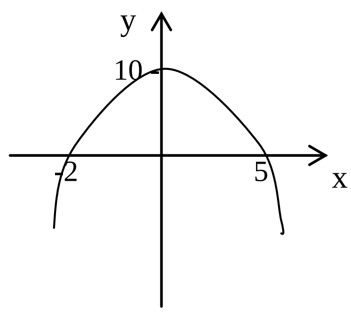
$y = -x^2 - 8x - 16.$

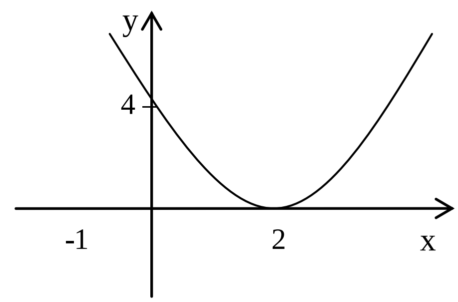
EVALUATION

Find the equations of the graph in the figure below:-









GENERAL EVALUATION

1. a. Draw the graph of $y = x^2 + 2x - 2$ from $x = -4$ to $x = +2$.
   b. Hence find the approximate roots of the equation $x^2 + 2x - 2 = 0$
2. a. Draw the graph of $y = x^2 - 5x + 6$ from $x = -5$ to $x = +1$

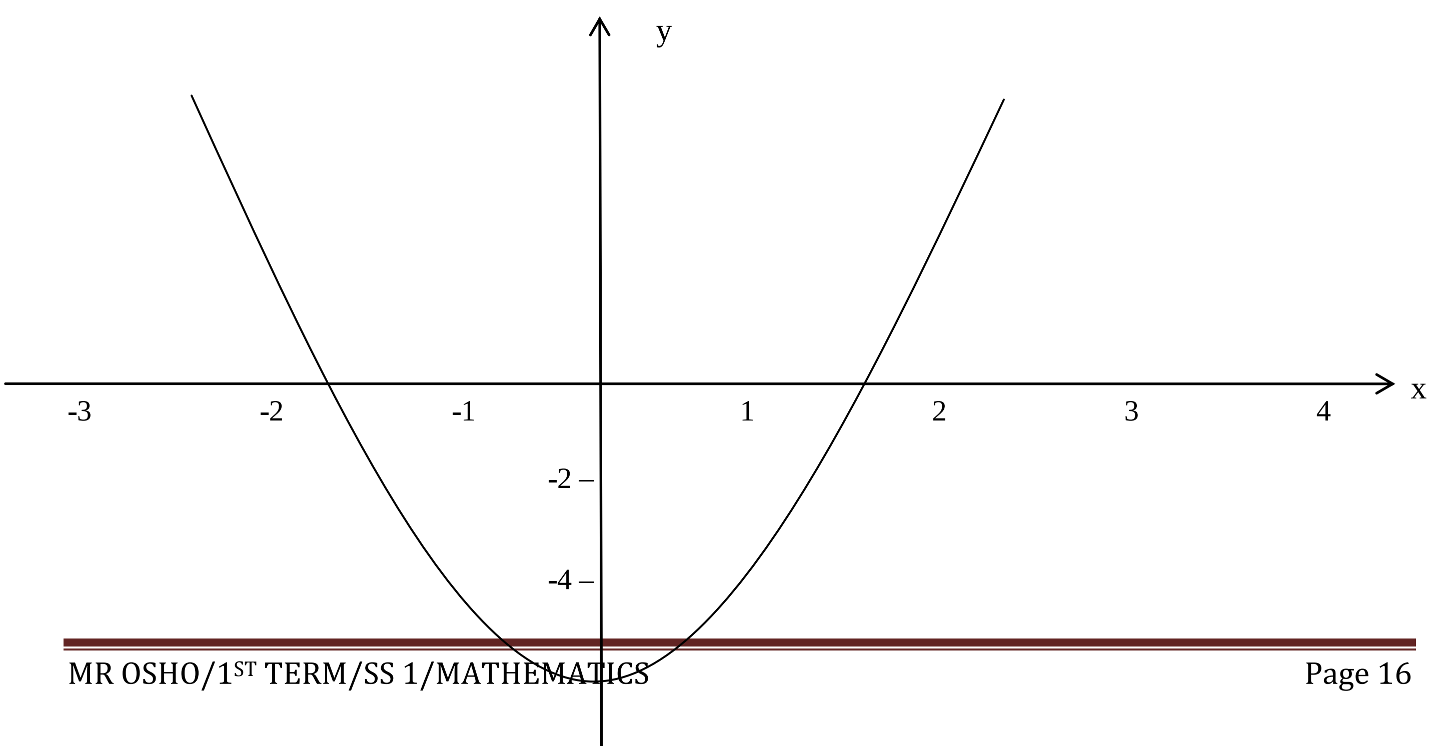
b. Hence find the approximate roots of the equation $x^2 - 5x + 6 = 0$

READING ASSIGNMENT

New General mathematics SS 1 pages 69– 74 by MF macrae et al

WEEKEND ASSIGNMENT

Use the graph below to answer question 1- 5



-6 –

1. Find the equation of the graph above
2. What are the solutions of the equations obtained in question (I) above?
3. What is the minimum value of y?
4. From the graph, what is the value of x when y = 2?
5. From the graph, what is the value of y when x = 1 ½ ?

THEORY

a. Prepare a table of values for the graph of $y = x^2 + 3x – 4$ for values of x from – 6 to + 3
b. Use a scale of 1cm to 1 unit on both axes and draw the graph.
c. Find the least value of y
d. What are the roots of the equation $x^2 + 3x – 4 = 0$?
e. Find the values of x when y = 1

WEEK FOUR
TOPIC: IDEA OF SETS
CONTENT

- Notation of Set
- Types and Operation of Set.

Definition of Set
A set is a welldefined collection of objects or elements having some common characteristic or properties. A set can be described by

I. Listing of its elements
II. Giving a property that clearly defines its element

Notations used in set theory

1. Elements of a set: the members of a set are called elementse.g list the elements of set

   A = { even numbers less than 10 }
2. n(A) means number of elements contained in a set
3. E means 'is an element of or 'belongs to' e.g 6EA
4. ∉ means 'is not an element of' or'did not belong to' e.g 5 ∉A defined in number 1 above
5. (:) means such that e.g B={X : 3 ≤ X ≤ 10} means X is a member of B such that X is a number from 3 to 10
6. Equal set: two sets are equal if they contain the same elements e.gIf S = {a,d,c,b} and P= {b,a,d,c,a,b}, then S=P repeated elements are counted once
7. Φ or { } means empty set or null set i.e A set which has no element e.g {secondary school student with age 3}
8. ⊂means subset. B is a subset of A if all the elements of B are contained in A e.gIf A ={1,2,3,4} and B = {1,2,3} then B is a subset of A i.e B ⊂A
9. U means union: all elements belonging to two or more given sets. A U B means list all elements in A and B e.g.If A ={2,4,6,8,10} and B = {1,3,5,7,9} then A U B ={1,2,3,4,5,6,7,8,9,10}
10. ∩ means intersection i.e elements common to 2 or more sets e.gA ={1,2,3,4,5,6} and B ={1,3,5,7,9} then A∩B = {1,3,5}
11. Ʊ and E means universal set i.e a large set containing all the original given set i.e A set containing all elements in a given problem or situations under consideration
12. Complement of a set i.e $A^{|}$. $A^{|}$ means 'A complement' and it is the set which contains elements that are not elements of set A but are in the universal set under consideration. E.gIf E ={shoes and sock} and A={socks}, then $A^{|}$ ={shoes}

EVALUATION

1. State the elements in the given set below: Y= {Y: Y E integer -4≤Y≤ 3}
2. Let E={x÷10<x< 20}   P= {prime numbers}   Q= {odd numbers}
   Where P and Q are subsets of E

a) List all elements of set P (b) What is n(P)? (c) List all elements of set Q (d) List the elements of $P^{|}$

3. Make each of the following statements true by writing Ɇ or E in place of *

a) 17 * 1,2,3,………7, 8,9 { }

b) 11 * 1,3,5,7…………. 19 { }

TYPES OF SETS

1. Universal set: A larger set containing all other sets under consideration i.e a set of students in a school
2. Finite set: is a set which contains a fixed number of elements. This means that a finite set has an end. E.g B={1,2,3,4,5}
3. Infinite set: is a set which has unending number of elements or which has an infinite number of elements. An infinite set has no end of its elements. E.g D={5,10,15,20…………….}
4. Subset: B is a subset of A if all elements of B are contained in Ai.e it is a smaller set contained in a larger or bigger set. E.g if A = {1,2,3,4,5,6} and B= {2,3,6} then B is a subset of A i.e B ⊂ A
5. Empty set Φ or { }. An empty set or null set contains no element
6. Disjoint set: if two sets have no elements in common, then they are said to be disjoint e.g If P= {2,5,7} and Q= {3,6,8} then P and Q are disjoint.

OPERATIONS OF SET

1. Intersection ∩: the intersection of two sets A and B is the set containing the elements common to A and B e.g if A= {a,b,c,d,e} and B= {b,c,e,f}, then A ∩ B= {b,c,e}
2. Union ∪: the union of A and B, A ∪ B is a set which includes all elements of A and B e.g if A = {1,3} and B = {1,2,3,4,6}, then A ∪ B ={1,2,3,4,6}
3. Complement of a set: the complement of a set P, $P^{|}$ are elements of the universal set that that are not in P e.g if U = {1,2,3,4,5,6} P= {2,4,5,6}, then $P^{|}$= {1,3}

Examples

Given that U = {a,b,c ,d,e,f}, P={b,d,e} Q= {b,c,e,f}

List the elements of

a) P∩ Q (b) P ∪ Q (c) $(P \cap Q)^{|}$

(d)$(P \cup Q)^{|}$ (e) $P^{|}$∪ Q (f)$Q^{|}$∩ $P^{|}$

Solution

a) P∩ Q = {b,e}

b) P ∪ Q= {b, c, d, e, f}

c) Since (P ∩ Q) = {b, e}
Then $(P \cap Q)^{|}$ = {a, c, d, f}

d) Sine (P ∪ Q)= {b, c, d, e, f}, then $(P \cup Q)^{|}$ ={a}

e) $P^{|}$∪ Q
$P^{|}$ ={a, c, f}
Q={b, c, e, f}
Therefore $P^{|}$∪ Q={a, b, c, e, f}

f) $Q^{|}=\{a, d\}$
$P^{|}=\{b, d, e\}$ $= P^{|}\cap Q^{|} = \{d\}$

EVALUATION
Given that U= {1,2,3,4,5,6,7,8,9,10}, A= {2,4,6,8} B= {1,2,5,9} and C= {2,3,9,10}
Find: a) $A\cap B\cap C$ (b) $C^{|}\cap(A\cap B)$ (c) $C\cap(A\cap B)^{|}$ (d) $C^{|}\cup(A\cap B)$

GENERAL EVALUATION
1. Given that U= {1,2,3…………19,20} and A ={1,2,4,9,19,20} B= {perfect square} C={factors of 24}. Where A,B, and C are subsets of universal set U
   a) List all the elements of all the given sets
   b) Find (i) $n(A \cup B)|$ (ii) $n(A \cup B \cup C)$ (iii) $n(A^{|}\cup B^{|}\cap C)$
   c) Find (i) $A\cap B\cap C$ (ii) $A\cup(B \cap C)$ (iii) $(A^{|}\cap B^{|})\cup C$
2. List all the subsets of the following sets
   a) A={Knife, Fork}
   b) P={a, e, i}

READING ASSIGNMENT
NGM SSS1 page 71-72, exercise 5b and 5c.

WEEKEND ASSIGNMENT
1. If A={a, b, c} B={a, b, c, e} and C={a, b, c, d, e, f} find $A\cap B(A\cup C)$ A.{a,b,c,d} B. {a,b,c,d,e} C.{a,b,d,d,e} D.{a,b,c}
2. If Q={0<x<30,x is a perfect square}, P={x÷1≤x≤10,x is an odd number} find $Q\cap P$ A.{1,3,9} B.{1,9,4} C.{1,9} D.{19,16,25}

Use the following information to answer questions 3 – 5
A,B and C are subsets of universal set U such that U={0,1,2,3……..11,12}, A={x:0<x<7}, B={4,6,8,10}, C={1<x<8}

3. Find $(A\cup C)^{|}$ A{0,1,9} B.{2,3,4,5} C.{2,3,5,7} D.{0,1,2,9}
4. Find $A|\cap B \cap C$
5. $A \cup B^{|}\cap C$ A.{1,2,3,4,5,6,7} B.{2,3,5,7} C.{6,8,10,12} D.{4,5,7,9,11}

THEORY
1. The universal set U is the set of integers: A,B and C are subsets of U defined as follows
   A= {….., -6,-4,-2,0,2,4,6…….}
   B= {X: 0 <x < 9}
   C= {X: -4 < x < 0}
   a) Write down the set A^{I}, where A^{I} is the complement of A with respect to U
   b) Find $B\cap C$
   c) Find the members of set $B\cup C$, $A\cap B$, and hence show that $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
2. The universal set U is the set of all integers and the subsets P,Q,R of U are given by
   P={X: X<0}, Q = {……,-5-,3,-1,1,3,5…….}, R= {X: -2<X<7}
   a) Find $Q\cap R$

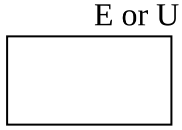
b) Find $R^{|}$ where $R^{|}$ is the complement of R with respect to U
c) Find $P^{|} \cap R^{|}$
d) List the members of $(P \cap Q)$

WEEK FIVE
TOPIC: SETS
CONTENT

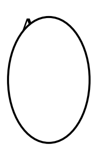
- Venn Diagram and Venn diagram Representation.
- Using of Venn Diagram to Solve Problems Involving Two Sets.
- Using Venn Diagram to Solve Problems Involving Three Sets.

THE VENN DIAGRAM
The venn diagram is a geometric representation of sets using diagrams which shows different relationship between sets
Venn diagram representation

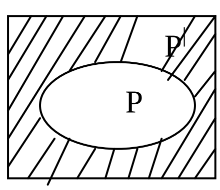
E or U



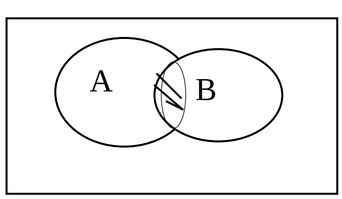
The rectangle represents the universal set i.e E or U



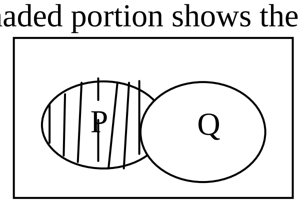
The oval shape represents the subset A.



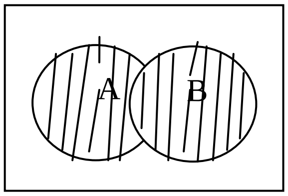
The shaded portion represents the complement of set P i.e $P^{|}$ or P^{c}



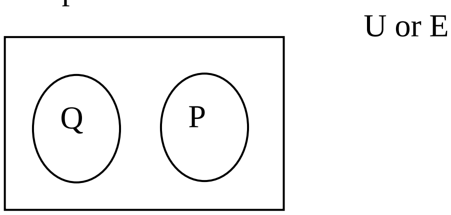
The shaded portion shows the elements common to A and B i.e $A \cap B$ or A intersection B.



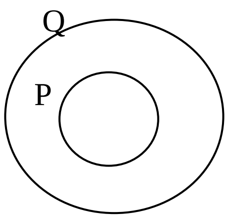
The shades portion shows P intersection $Q^{|}$i.e $P \cap Q^{|}$



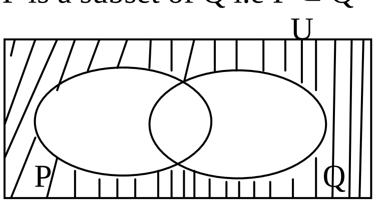
The shaded portion shows A∪ B i.e A union B



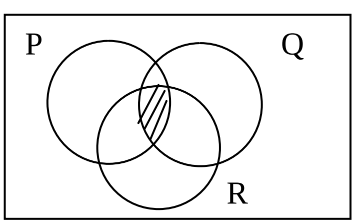
This shows that P and Q have no common element. i.e P and Q are disjoint sets i.e $P \cap Q = \Phi$



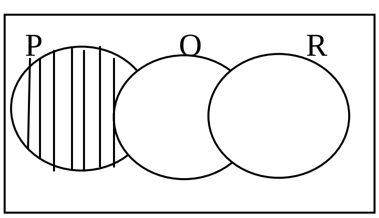
P is a subset of Q i.e $P \subset Q$



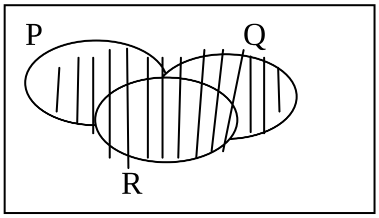
$P^I \cap Q^I$ or $(P \cup Q)|^I$. This shows elements that are neither in P nor Q but are represented in the universal set.



This shows the elements common to set P,Q and R i.e the intersection of three sets P,Q and R i.e $P \cap Q \cap R$



This shows the elements in P only, but not in Q and R i.e $P \cap Q^{|} \cap R^{|}$



This shaded region shows the union of the three sets i.e $P \cup Q \cup R$

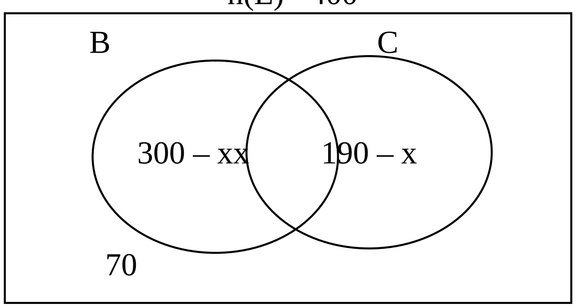
USING THE VENN DIAGRAM TO SOLVE PROBLEMS INVOVING TWO SETS

Examples:

1. Out of 400 final year students in a secondary school, 300 are offering Biology and 190 are offering Chemistry. If only 70 students are offering neither Biology nor Chemistry. How many students are offering (i) both Biology and Chemistry? (ii) At least one of Biology or Chemistry?

Solution

n(E)= 400



Let the number of students who offered both Biology and Chemistry be X i.e $(B\cap C)$= X. from the information given in the question

n(E)= 400

n(B)= 300

n(C)= 190

$n(B\cup C)'$= 70

Since the sum of the number of elements in all region is equal to the total number of elements in the universal set, then:

300 - x + x +190 – x + 70 =400

560 – x= 400

-x= 400 – 560

X= 160

Number of students offering both Biology and Chemistry= 160

(ii)Number of students offering at least one of Biology and Chemistry from the Venn diagram includes those who offered biology only, chemistry only and those whose offered both i.e

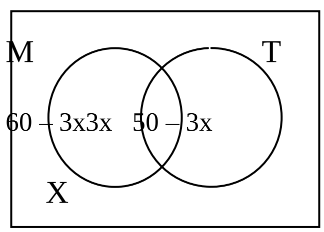
300 – x + 190 – x + x= 490 - x

490 – 160 (from (i) above) = 330

2. In a youth club with 94 members, 60 likes modern music and 50 likes traditional music. The number of them who like both traditional and modern music are three times those who do not like any type of music. How many members like only one type of music

Solution
Let the members who do not like any type of music = X
Then,
n(T n M)= 3X
Also,
n(E)= 94
n(M)=60
n(T)= 50
n(M u T)$^{|}$= X

n(E)= 94



Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then
60 – 3X + 3X + 50 – 3X + X = 94
110 – 2X= 94
16= 2X
Divide both sides by 2
$\frac{16}{2} = \frac{2X}{2}$
X= 8
Therefore number of members who like only one type of music are those who like modern music only + those who like traditional music only.
60 -3x + 50 – 3x
110 – 6x
= 110 – 6(8) = 110 - 48
= 62

EVALUATION
1. Two questions A and B were given to 50 students as class work.23 of them could answer question A but not B. 15 of them could answer B but not A. If 2x of them could answer none of the two questions and 2 could answer both questions.
   a) Represent the information in a Venn diagram.
   b) Find the value of x
2. In a class of 50 pupils, 24 like oranges, 23 like apples and 7 like the two fruits.

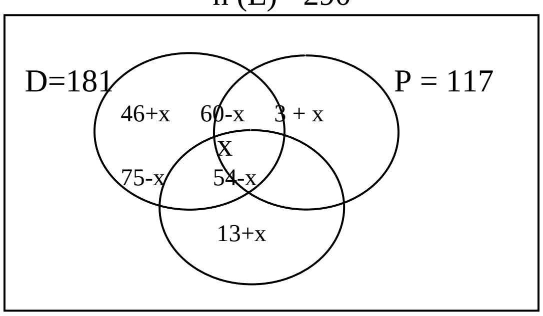
a) How many do not like oranges and apples
b) What percentage of the class like apples only

USING VENN DIAGRAM TO SOLVE PROBLEMS INVOLVING THREE SETS
Examples:

1. In a survey of 290 newspaper readers, 181 of them read the Daily Times, 142 read the Guardian, 117 read the Punch and each read at least one of the papers, If 75 read the Daily Times and the Guardian,60 read the Daily Times and Punch and 54 read the Guardian and the Punch.
   a) Draw a Venn diagram to illustrate the information
   b) How many read:
      (i) all the three papers.
      (ii) exactly two of the papers.
      (iii)exactly one of the papers.
      (iv)the Guardian only.

Solution

n (E)= 290



n(P)= 117
n(E)= 290
n(D)= 181
n(G)= 142
n(D∩G)= 75
n(D∩P)= 60
n(G∩P)= 54
From the Venn diagram, readers who read Daily Times only
=181 – (60 – X + 75 – X +X) = 181 – (135 - X) = 46 + X
Punch readers only = 117 – (60 – X + 54 – X + X) = 117 – (114 - X) = 117 – 114 + X
=3 +X

Guardian readers only
=142 – (75 – X + 54 – X + X)
=142 – (129 - X)
=142 – 129 + X
=13 + X
Where:

X is the number of readers who read all the three papers

Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then:

46 + X + 75 – X + 13 + X + 60 – X + X + 54 – X + 3 + X = 290

251 + X = 290

X = 290 – 251

X= 39

b(i): number of people who read all the three papers = 39

(ii) from the Venn diagram, number of people who read exactly two papers

= 60 – X + 75 – X + 54 – X

=189 – 3X = 189 – 3(39) from the above

=189 – 117 = 72

(iii) also, from the Venn diagram, number of people who read exactly only one of the papers

=46 + X + 13 + X + 3 +X

= 62 +3X = 62 + 3(39)

= 62 + 117 = 179

(iv)number of Guardian reader only

=13 + X

=13 + 39 = 52

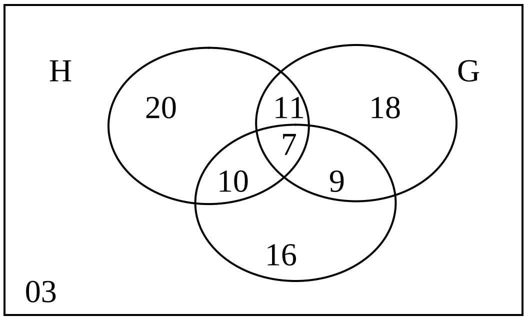
2. A group of students were asked whether they like History, Science or Geography. There responds are as follows:

Subject liked	**Number of students**
All three subjects	7
History and Geography	11
Geography and Science	09
History and Science	10
History only	20
Geography only	18
Science only	16
None of the three subjects	03

a) Represent the information in a Venn diagram
b) How many students were in the group?
c) How many students like exactly two subjects

Solution

a) n(E)= ?



b) Number of students in the group = sum of the elements in all the regions i.e
Number of students in the group = 20 + 18 + 16 + 11 + 9 + 10 + 7 + 3 = 94

c) Number of students who like exactly two subject = 11 + 9 + 10 = 30

Evaluation

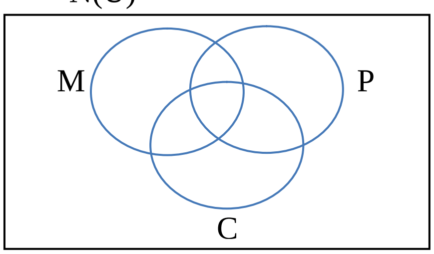
1. In a community of 160 people, 70 have cars ,82 have motorcycles, and 88 have bicycles: 20 have both cars and motorcycles,25 have both cars and bicycles, while 42 have both motorcycles and bicycles.Each person rode on at least any of the vehicles

a) Draw a Venn diagram to illustrate the information.

b) Find the number of people that has both cars and bicycles.

c) How many people have either one of the three vehicles?

2. N(U)



The score of 144 candidates who registered for Mathematics, Physics and Chemistry in an examination in a town are represented in the Venn diagram above.

a) How many candidate register for both Mathematics and Physics

b) How many candidate register for both Mathematics and Physics only

GENERAL EVALUATION

1. In a senior secondary school, 80 students play hockey or football. The numbers that play football is 5 more than twice the number that play hockey. If 5 students play both games and every students in the school plays at least one of the games. Find:

a) The number of students that play football

b) The number of students that play football but not hockey

c) The number of students that play hockey but not football

2. A, B and C are subsets of the universal set U such that

U={0,1,2,3,4…………12}
A={X: 0≤ x <7} B= {4,6,8,10,12} C= {1<y<8} where Y is a prime number.

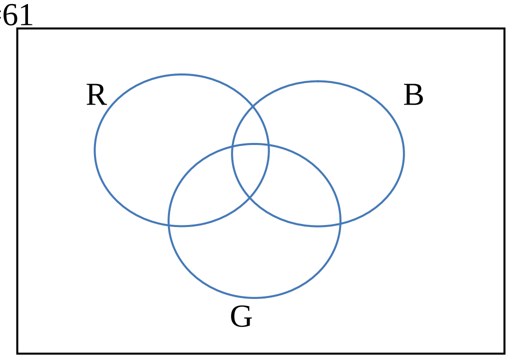
a) Draw a venn diagram to illustrate the information
b) Find (i) BỦC (ii) A∩B∩C

READING ASSIGNMENT
NGM SSS1,page 106, exercise 8d, numbers 11-17.

WEEKEND ASSIGNMENT

1. In a class of 50 pupils, 24 like oranges, 23 like apples and 7 like the two fruits. How many students do not like oranges and apples? (a)7 (b) 6 (c) 10 (d)15
2. In a survey of 55 pupils in a certain private school, 34 like biscuits, 26 like sweets and 5 of them like none. How many pupils like both biscuits and sweet? (a) 5(b) 7 (c)9 (d)10
3. In a class of 40 students, 25 speaks Hausa, 16 speaks Igbo, 21 speaks Yoruba and each of the students speaks at least one of the three languages. If 8 speaks Hausa and Igbo, 11 speaks Hausa and Yoruba,6speaks Igbo and Yoruba. How many students speak the three languages? (a) 3 (b) 4 (c) 5 (d) 6

Use the information to answer question 4 and 5
N(U)=61



The Venn diagram above shows the food items purchased by 85 people that visited a store in one week. Food items purchased from the store were rice, beans and gari.

4. How many of them purchased gari only? (a)8 (b)10 (c) 14 (d)12
5. How many of them purchased the three food items? (a) 5 (b)7 (c) 9 (d)11

THEORY

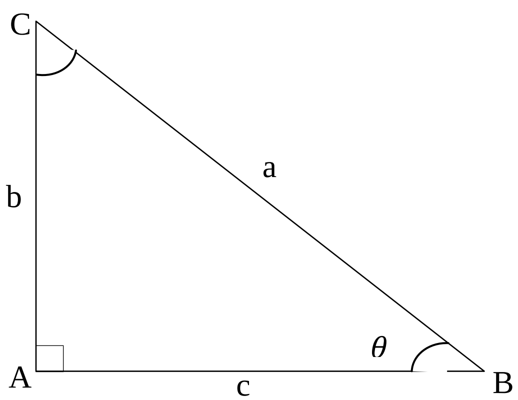
1. In a certain class, 22 pupils take one or more of Chemistry, Economics and Government. 12 take Economics (E), 8 take Government (G) and 7 take Chemistry (C). nobody takes Economics and Chemistry and 4 pupils takeEconomics and Government
   a) Using set notation and the letters indicated above, write down the two statements in the last sentence.
   b) Draw the Venn diagram to illustrate the information
2. How many pupils take
   a) Both Chemistry and Government?
   b) Government only?

WEEK SIX
Review of the first half term's work and periodic test

WEEK SEVEN
TOPIC: TRIGONOMETRIC RATIOS
CONTENT

- Sine,Cosine and Tangent of acute angles
- Use of tables of Trigonometric ratios
- Determination of lengths of chord using trigonometric ratios.
- Graph of sine and cosine for angles

Sine, Cosine and Tangent of acute angles
Given a right angled triangle, the trigonometric ratio of acute angles can be found as shown below



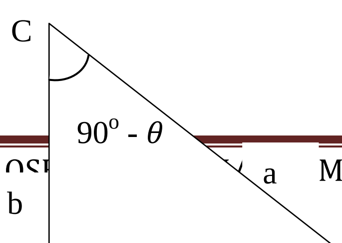
In the figure above, ABC is any triangle, right-angled at A

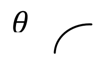
tan B = $\frac{b}{c}$ tan C = $\frac{c}{b}$(tan : $\frac{Opp}{Adj}$)

Sin B = $\frac{b}{a}$ Sin C = $\frac{c}{a}$ Sin ;$\frac{Opp}{Hyp}$

Cos B = $\frac{c}{a}$ Cos C = $\frac{b}{a}$ (Cos : $\frac{Adj}{Hyp}$)

In△ABC, B and C are complementary angles i.e B + C = 90^o
If B = Ɵ then C = 90^o - Ɵ.





In the figure above $\sin \Theta = \cos(90-\Theta) = \frac{b}{a}$

$\cos \Theta = \sin(90-\Theta) = \frac{c}{a}$

Note: Always remember SOH CAH TOA

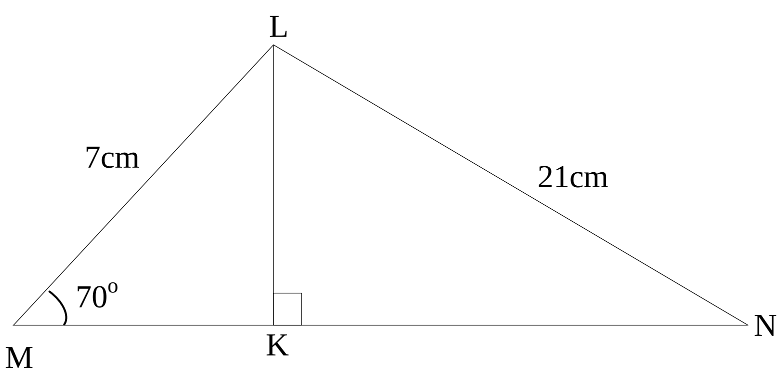
i.e $\sin \theta = \frac{Opp}{Hyp}$

$\cos \theta = \frac{Adj}{Hyp}$

$\tan O = \frac{Opp}{Adj.}$

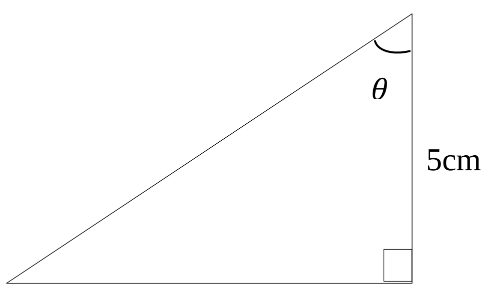
Examples

1. A triangle has sides 8cm and 5cm and an angle of 90^o between them .Calculate the smallest angle of the triangle
2. A town Y is 200km from town X in a direction 40^o. How far isY east of X ?
3. In the figure below, LK is perpendicular to MN. Calculate< MNL



Solutions

(1)



8cm

$\tan\theta = \dfrac{8cm}{5cm} = 1.600$

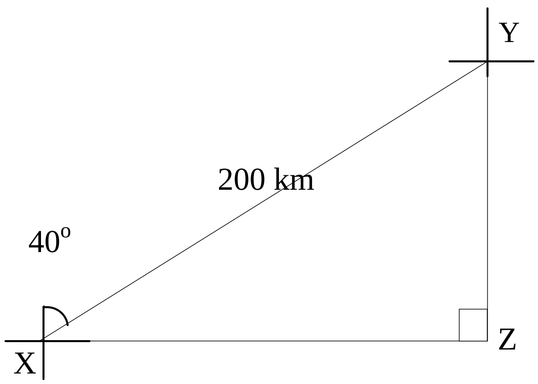
$\Theta = \tan^{-1} 1.6000$

$\Theta = 58^o$.

The 3rd angle in the right angled triangle above = $90^o - 58^o = 32$.

Hence, the smallest angle of the given triangle = 32^o

2)



From the diagram drawn above the distance of Y east of X = ZY

Using the right angled triangle XZY

$\sin 40^o = \dfrac{ZY}{200km}$

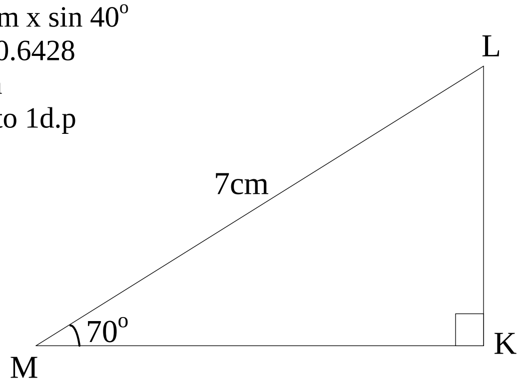
ZY = 200km x sin 40^o

= 200km x 0.6428

= 128.56km

= 128.6km to 1d.p

3)



In the given diagram,

$\dfrac{LK}{7cm} = \sin 70^o$

LK = 7cm x sin 70^o ………… i

LK = 7cm x 0.9397

LK = 6.5779cm

In right angled triangle LKN

$\frac{LK}{21}$ = sin MNL

i.e $\frac{7cm \times 0.9397}{21cm}$ = Sin MNL

$\frac{0.9397}{3}$ = Sin MNL

0.3132 = Sin MNL

$Sin^{-1}0.3132$ = MNL

18. 290 = MNL

i.e MNL = 18.3^{o}

EVALUATION

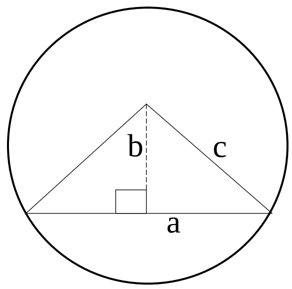
A ladder 20cm long rests against a vertical wall so that the foot of the ladder is 9m from the wall.

(a) Find, correct to the nearest degree, the angle that the ladder makes with the wall

(b) Find correct to 1.dp the height above the ground at which the upper end of the ladder touches the wall. Use of tables of trigonometric ratios.

Determination of lengths of chords using trigonometric ratios

Trignometric ratios can be used to find the length of chords of a given circle. However, in some cases where angles are not given, Pythagoras theorem is used to find the lengths of chords in such cases. Pythagoras theorem is stated as follows:

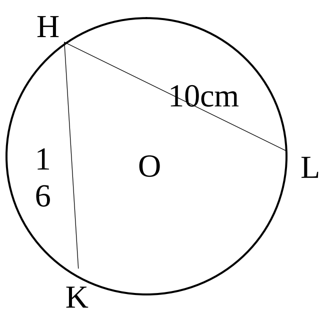


It states that $c^2 = a^2 + b^2$

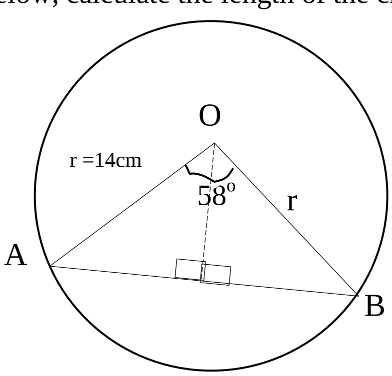
Pythagoras theorem states that in a right angled triangle, the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the other two sides.

Examples

1. A chord is drawn 3cm away from the centre of a circle of radius 5cm. Calculate the length of the chord.
2. In the figure below, O is the centre of circle, HKL. HK = 16cm, HL = 10cm and the perpendicular from O to the HK is 4cm. What is the length of the perpendicular from O to HL?

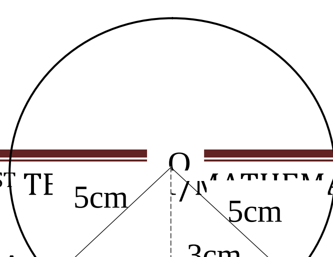


3. Given the figure below, calculate the length of the chord AB.



Solutions

1



B

From the diagram above in right-angled triangle ABO:

$|AB|^2 + 3^2 = 5^2$ (Pythagoras theorem)

$|AB|^2 = 5^2 - 3^2$

$|AB|^2 = 25 - 9$

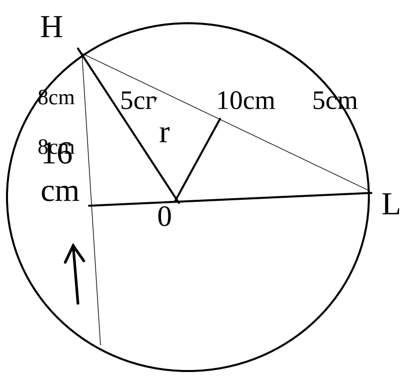
$|AB|^2 = 16$

$AB = \sqrt{16} = 4cm$

Since B is the mid point of chord AC then

Length of chord AC = 2 x AB
= 2x 4cm =8cm

2)



Let the distance from O to HL= xcm
In right-angled triangle OMH:

$|OH|^2 = |HM|^2 + |MO|^2$

$|OH|^2 = 8^2 + 4^2$

= 64 + 16
= 80
:. $|OH| = \sqrt{80}$

:. $|OH| = \sqrt{80}$cm

but $|OH|$ = radius of the circle

i.e $r = |OH| = |OL| = \sqrt{80}$cm

In right-angled triangle ONL

$|OL|^2 = |ON|^2 + |NL|^2$

i.e $(\sqrt{80})^2 = x^2 + 5^2$

$80 - 25 = x^2$

$55 = x^2$

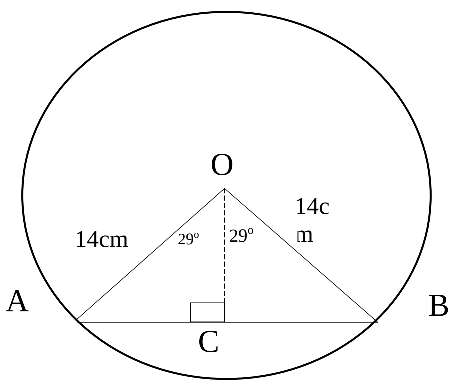
Take square root of both sides

$\sqrt{55} = \sqrt{x^2}$

$\sqrt{55} = x = 7.416$cm

:. The length of the perpendicular from O to HL is 7.416cm

3)



The perpendicular from O to AB divides the vertical angle into 2 equal parts and also divides the length of chord AB into two equal parts.

In right-angled triangle ACO:

$\frac{|AC|}{|OA|} = \frac{\sin 29^o}{1}$

Cross multiply

AC = OA x sin 29^o

AC = 14cm x Sin 29^o

AC = 14cm x 0. 4848

AC = 6.787cm

AB = 2 x 6.787 cm

AB = 13.574cm

:. The length of the chord AB = 13.6cm to 1 d.p

EVALUATION

1. A chord 30cm long is 20cm from the centre of a circle . Calculate the length of the chord which is 24cm from the centre .
2. Q is 1.4km from P on a bearing 023^o. R is 4.4 Km from P on a bearing 113^o. Make a sketch of the positions of P, Q and R and hence, calculate QR correct to 2 s.f.

GRAPH OF SINE AND COSINE FOR ANGLES

In the figure below, a circle has been drawn on a Cartesian plane so that its radius, $\overline{OP}$, is of length 1unit. Such a circle is called **unit circle.**

The angle Θ that $\overline{OP}$ makes with Ox changes according to the position of P on the circumference of the unit circle. Since P is the point (x,y) and /OP/ = 1 unit,

$\text{Sin } \Theta = y/1 \quad = y$

$\text{Cos } \Theta = x/1 \quad = x$

Hence the values of x and y give a measure of cos Θ and sin Θ respectively.

If the values of Θ are taken from the unit circle, they can be used to draw the graph of sin Θ. This is done by plotting values of y against corresponding values of Θ as in the figure below.

In the figure above, the vertical dotted lines gives the values of sin Θ corresponding to $\Theta = 30^o, 60^o, 90^o, \ldots\ldots, 360^o$.

To draw the graph of cosΘ , use corresponding values of x and Θ. This gives another wave-shaped curve, the graph of cos Θ as in the figure below.

As Θ increases beyond 360^o, both curves begin to repeat themselves as in the figures below.

Take note of the following:
1)All values of sin Θ and cos Θ lie between +1 and -1.
2)The sine and cosine curves have the same shapes but different starting points.
3)Each curve is symmetrical about its peak(high point) and trough(low point). This means that for any value of sin Θ there are usually two angles between 0^{o} and 360^{o}; likewise for cos Θ. The only exceptions to this are at the quarter turns, where sinΘ and cosΘ have the values given in the table below

	0^{o}	90^{o}	180^{o}	270^{o}	360^{o}
SinΘ	0	1	0	-1	0
CosΘ	1	0	-1	0	1

Examples
1) Referring to graph on page 194 0f NGM Book 1, a)Find the value of sin 252^{o}, b)solve the equation 5 sin Θ = 4
Solution
a)On the Θ axis, each small square represents 6. From construction a) on the graph:
Sin 252^{o} = -0.95
b)If 5 sin Θ = 4
then sin Θ = 4/5 = 0.8
From construction b) on the graph: when sin Θ = 0.8, Θ = 54^{o} or 126^{o}

EVALUATION
1)Using the same graph used in the above example, find the values of the following
a)sin 24^{o} b) sin 294^{o}
2)Use the same graph to find the angles whose sines are as follows:
a) 0.65 b)-0.15

GENERAL EVALUATION
a. Express the following in terms of sin, cos or tan of an acute angle:
   i. sin 210^{0}
   ii. tan 240^{0}
   iii. cos(-35^{0})
b. If cosθ = -0.6428, find the value of θbetween 0^{0} and 360^{0}

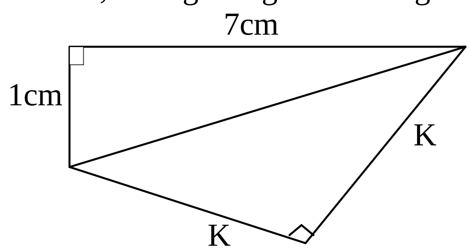
READING ASSIGNMENT
NGM SS BK 1 pg 114- 123, Ex 11a .
NOS 10 and 25 pg 117 -118

WEEKEND ASSIGNMENT
1. If Sin A = 4/5, what is tan A? A 2/5 B. 3/5 C. ¾D. 1 E. 4/3
2. Use tables to find the value of 8 Cos 77. A. 5.44 B. 6.48 C. 9.12 D 7.57 E. 1.80
3. If cos θ = sin 33 ^{o}, find tan θ. A. 1.540 B. 2.64 C.0.64 D. 1.16 E. 1.32

4. If the diagonal of a square is 8cm, what is the area of the square? A $16cm^2$B. $2cm^2$ C. $4cm^2$D. $20cm^2$E. $10cm^2$
5. Calculate the angle which the diagonal in question 4 makes with any of the side of the square. A. 65^oB. 45^oC. 35^oD. 25^oE. 75^o

THEORY

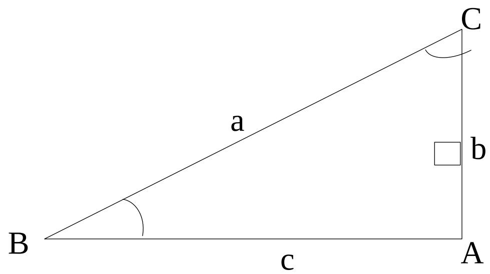
1) From a place 400m north of X, a student walks east wards to a place Y which is 800m from X. What is the bearing of X from Y?
2) In the figure below, the right angles and lengths of sides are as shown. Calculate the value of K .



WEEK EIGHT

- Application of sine, cosine and tangent, simple problems with respect to right angle triangles.
- Angles of elevation and depression
- Bearing and distances of places strictly by application of trigonometric ratio.

(a)



In the figure above,triangle ABC is any triangle, right-angled at A.

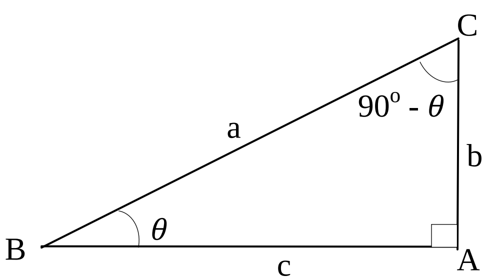
tan B = b/c , tan C = c/b (tan:opp/adj.)

sin B = b/a , sin C = c/a (sin:opp/hyp.)

Cos B = c/a , cos C = b/a (cos:adj./hyp.)

In $\triangle$ABC, B and C are complementary angles(i.e. B + C = 90).

If B = ϴ, then C= 90^o – ϴ (as in below)

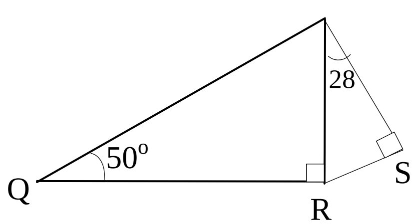


Sin ϴ = Cos (90^o – ϴ) = b/a and

Cosϴ = sin (90^o – ϴ) = c/a

Examples

1)Calculate a)/PR/, b)/RS/ in the figure below.Give the answers correct to 3 s.f.



Solution

a)In triangle PQR,

tan 50^o = x/6

x = 6 X tan 50^o= 6 X 1.192

= 7.152

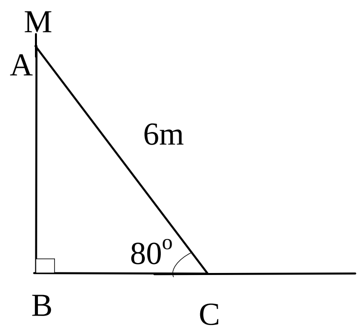
/PR/ = 7.15 cm to 3 s.f.

a) In triangle PRS,

Sin 28^o = y/x
y = x sin 28 = 7.152 x 0.4695 = 3.358
/RS/ = 3.36 cm to 3 s.f.

2) A ladder of length 6.0 m rests with its foot on a horizontal ground and leans against a vertical wall.The inclination of the ladder to the horizontal is 80^o. Find correct to one decimal place
a) the distance of the foot of the ladder from the wall, b) the height above the ground at which the upper end of the ladder touches the wall.

Solution



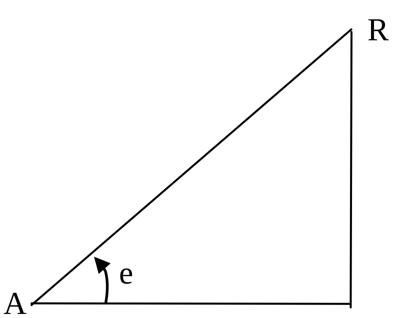
In the figure above,
AC is the ladder
MB is the wall
ACB is the inclination of the ladder to the horizontal = 80^o

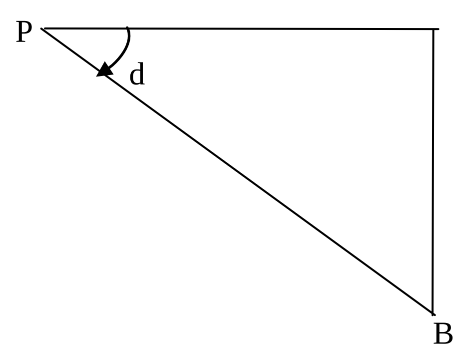
a) The distance of the foot of the ladder from the wall is BC, where
Cos 80^o = BC/6
BC = 6 Cos 80^o = 6 X 0.1736 = 1.0416 m
= 1.0 m to 1 d.p.

b) The height above the ground at which the upper end of the ladder touches the wall is AB, where
Sin 80^o = AB/6
AB = 6 Sin 80^o = 6 x 0.9848 = 5.9088 m
= 5.9 m to 1 d.p.

(a) Angle of elevation and depression

The figure below shows the angle of elevation e , of the top of the tower, R , from a point A below. The diagram also shows the angle of depression, d , of a point B on the ground from a point, p , on the tower.

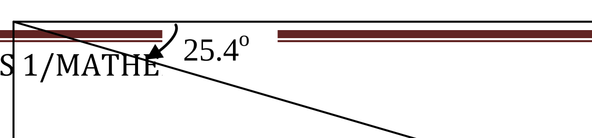


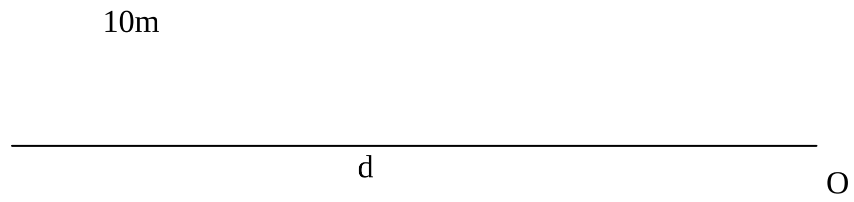


Examples

1)From a window 10m above level ground , the angle of depression of an object on the ground is 25.4^o.Calculate the distance of the object from the foot of the building.

Solution





$\tan 25.4^o = 10/d$

$d = 10/\tan 25.4^o$

$= 10/0.4748$

$= 21.06m$

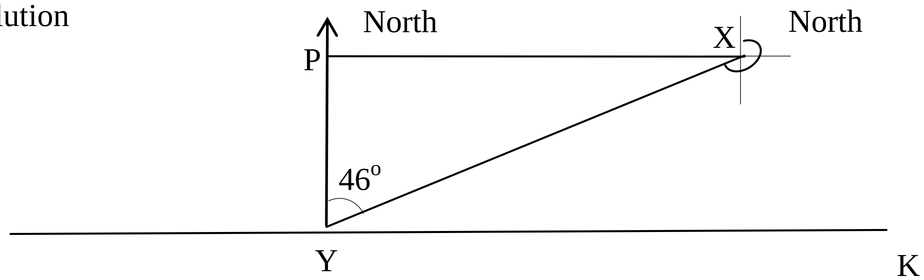
The object is 21.06m from the foot of the building

Bearing and Distances

Examples

1)The bearing of X from Y is 046^o. What is the bearing of Y from X ?

Solution



The bearing of X from Y is XYP = 046^O

The bearing of Y from X is reflex XY = ϴ

ϴ =180 + 46 = 226^o

EVALUATION

1)The angle of elevation from the top of a tower from a point on the horizontal ground, 40m away from the foot of the tower , is 30^o. Calculate the height of the tower to two significant figures.

2)From the top of a light-tower 40m above sea level, a ship is observed at an angle of depression of 6^o. Calculate the distance of the ship from the foot of the light-tower, correct to 2s.f.

3)From a point P, R is 8km due east and 8km due south. Find the bearing of P from R

GENERAL EVALUATION

1. Express the sine, cosine and tangent of (a) 30^0, (b) 150^0, (c) 210^0, (d) 330^0 as either a positive or a negative trigonometrical ratio of an acute angle.

READING ASSIGNMENT

NGM BK 1 PG 114 – 129; Ex 11e nos 1 - 10

WEEKEND ASSIGNMENT

1)A town Y is 200 Km from town X in a direction 040^o. How far is Y east of X?

a)125.8km b)128.6km c)127km d)126.8km

2)A boy walks 1260m on a bearing of 120^o. How far Southis he from his starting point?

a)630m b)530m c)730m d)630km

Use the figures below to answer questions 3-5

Calculate to 2 s.f., the values of
3)w a)0.21 b)21 c)2.1 d)2.31
4)x a)5 b)8 c)9 d)10
5)y a)6.5 b)7.5 c)8.5 d)9.5

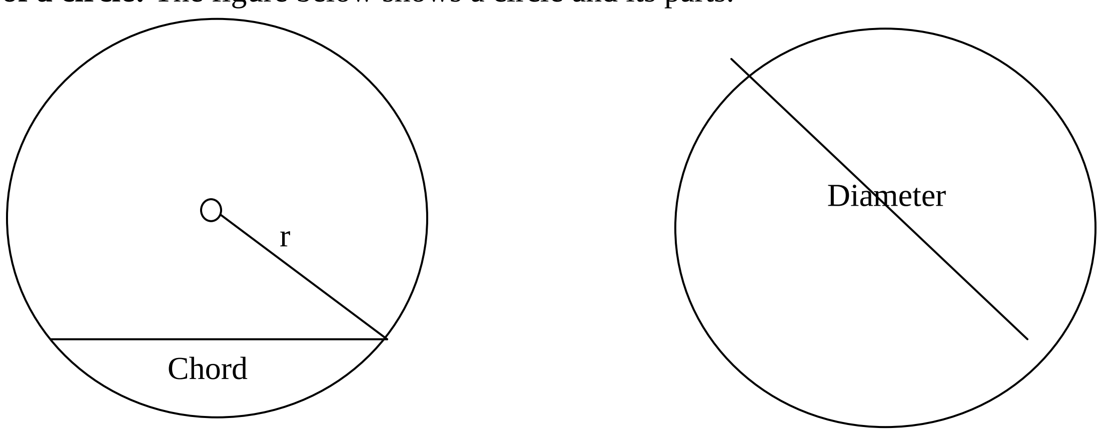
THEORY

1. A rhombus has sides 11 cm long. The shorter diagonal of the rhombus is 8cm long. Find the size of one of the smaller angles of the rhombus correct to the nearest degree.
2. a)Fron the top of a cliff, the angle of depression of a boat on the sea is 22^{o}.If the height height of the cliff above sea level is 40m, calculate, correct to 2 significant figures, the distance of the boat from the bottom of the cliff.
   b)At a point 20m from the base of a water tank, the angle of elevation of the top of the tank is 45^{o}. What is the height of the tank?

WEEK NINE
TOPIC

- Introduction of circle and its properties
- Calculation of length of arc and perimeter of a sector
- Area of sectors and segments. Area of triangles

(a) Introduction of circle and its properties
Parts of a circle: The figure below shows a circle and its parts.

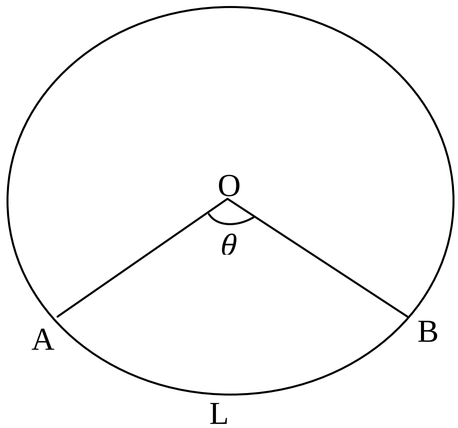


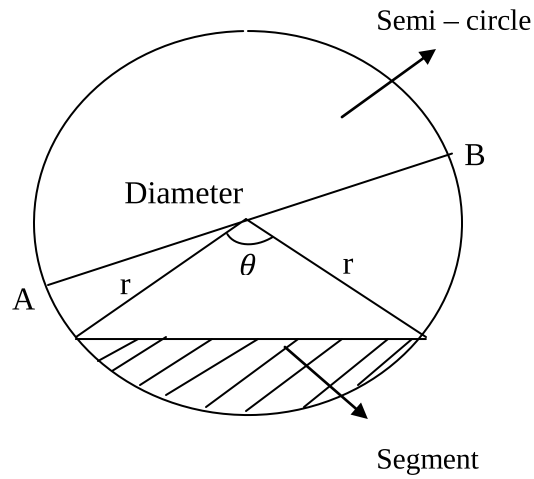
The centre is the point at the middle of a circle. The circumference is the curved outer boundary of the circle. An arc is a curved part of the circumference. A radius is any straight line joining the centre to the circumference. The plural of radius is radii. A chord is any straight line joining two points on the circumference. A diameter is a straight line which divides the circle into two equal parts or a diameter is any chord which goes through the centre of the circle.

Region of a circle

The figure below shows a circle and its different regions.

A sector is the region between two radii and the circumference. A semi-circle is a region between a diameter and the





circumference i.e half of the circle. A segment is the region between a chord and the circumference.

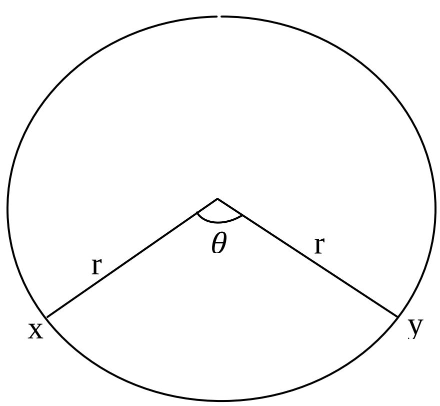
EVALUATION

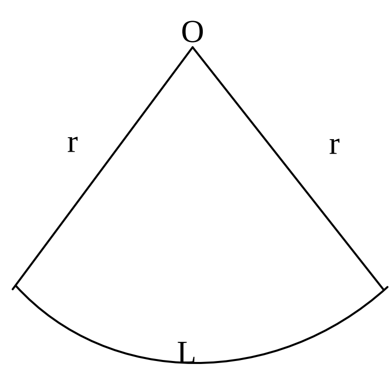
Draw a circle and show the following parts on it. Two radii, a sector, a chord, a segment, a diameter, an arc; label each part and shade any regions.

(b) Calculation of length of arc and perimeter of a sector

Given a circle centre O with radius r. The circumference of the circle is 2Πr. Therefore, in the figure below, the length, L, of arc XY is given as:

$$L = \frac{\theta \times}{360^o} 2\Pi r$$





Where θ is the angle subtended at the centre by arc XY and r is the radius of the circle.

Also,

The perimeter of Sector XOY = r + r + L

Where

L = length of arc XY

$= \frac{\theta}{360} \times 2\Pi r$

Then

Perimeter of Sector XOY $= r + r + L$

$= 2r + \frac{\theta}{360^o} \times 2\Pi r$

EXAMPLES

1. An arc of length 28cm subtends an angle of 24^0 at the centre of a circle. In the same circle, what angle does an arc of length 35cm subtend?
2. Calculate the perimeter of a sector of a circle of radius 7cm, the angle of the sector being 108^o, if Π is $\frac{22}{7}$.

Solutions

1. $L = \frac{\theta}{360} \times 2\Pi r$

When L = 28cm , $\theta = 24^0$, r = ?

Then

$$L = \frac{\theta}{360^o} \times 2\Pi r$$

$$28 = \frac{24}{360^0} \times 2 \times \frac{22}{7} \times r$$

Cross-multiply:

$24 \times 44 \times r = 28 \times 360 \times 7$

$$r = \frac{\overset{7}{\cancel{28}} \times \overset{\overset{15}{\cancel{60}}}{\cancel{360}} \times 7\ cm}{\underset{\cancel{4}}{\cancel{24}} \times \underset{11}{\cancel{44}}}$$

$$r = \frac{49 \times 15\ cm}{11}$$

$r = \frac{735}{11}$ cm

Also

When L = 35cm, $r = \frac{735}{11}$ cm

$\theta = ?$

Then

$$L = \frac{\theta}{360^o} \times 2\Pi r$$

$$35 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times \frac{735}{11}$$

Then,

Cross multiply

$$\frac{35}{1} \times 360 \times 7 \times 11 = \theta \times 44 \times 735$$

$$\frac{\cancel{35}^{1} \times 360 \times \cancel{77}^{11}}{\cancel{44}_{4} \times \cancel{735}_{\cancel{105}\,3}} = \theta$$

$$\frac{360}{12} = \theta = 30^0$$

Thus, when the length of the arc is 35cm, the angle subtended at the centre is 30^0

2. Perimeter of a sector of a circle $= 2r + \frac{\theta}{360^o} \times 2\Pi r$

$$= 2 \times 7 + \frac{108}{360} \times 2 \times \frac{22}{\cancel{7}_{1}} \times \cancel{7}$$

$$= 14 + \frac{\cancel{108}^{3} \times 44}{\cancel{360}_{10}} \text{cm}$$

$$= 14 + \frac{3 \times 44}{10} \text{ cm}$$

$$= 14 + \frac{132}{10} \text{ cm}$$

$$= 14 + 13.2 \text{ cm}$$

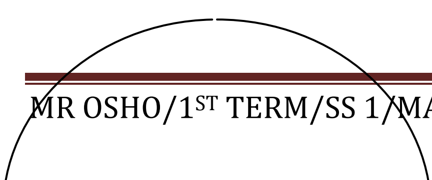
$$= 27.2 \text{ cm}$$

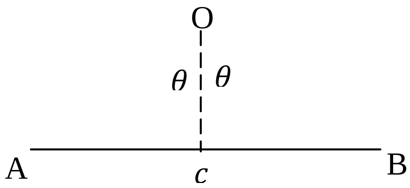
EVALUATION

1. A piece of wire 22cm long is sent into an arc of a circle of radius 4 cm. What angle does the wire subtend at the centre of the circle?
2. Calculate the perimeter of a sector of a circle of radius 3.5cm, the angle of the sector being 162^0 if Π is $\frac{22}{7}$.

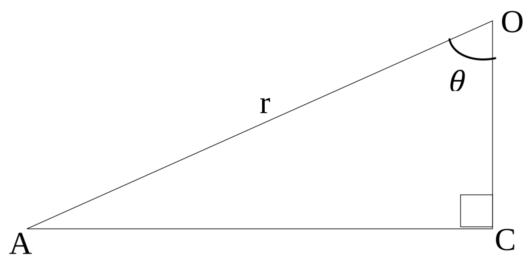
Length of chord and perimeter of a segment.

Consider a circle centre O with radius r





If OC is the perpendicular distance from O to chord AB and angle AOB = 2 θ, then the length of chord AB can be found as follows:



In right-angled triangle OCA

$$\frac{AC}{r} = \sin\theta$$

Cross multiply:

$AC = r \sin\theta$

Since

$AB = 2 \times AC$

$AB = 2r \sin\theta$

Where

r = radius of the circle

θ =Semi Vertical angle of the sector i.e half of the angle subtended at the centre by arc AB.

Also

The perimeter of segment ACBD = Length of chord AB + length of arc ADB

$$= 2r \sin\theta + \frac{\theta}{360^o} \times 2\Pi r$$

Example

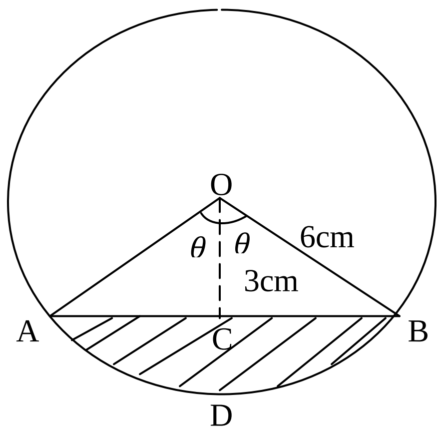
In a circle of radius 6 cm, a chord is drawn 3cm from the centre.

(a) Calculate the angle subtended by the chord at the centre of the circle.

(b) Find the length of the minor arc cut off by the chord

(b) Hence find the perimeter of the minor segment formed by the chord and the minor arc.

Solution



a. Let the required angle

$= AOB = 2\theta$

Where

θ = Semi vertical angle of the sector.

Then

$\text{Cos } \theta = \frac{3cm}{6cm} = \frac{1}{2}$

$\text{Cos } \theta = 0.5000$

$\theta = \text{Cos}^{-1}\ 0.5000$

$\theta = 60^0$

-: Required angle $= 2\theta$

$= 2 \times 60^0$

$= 120^0$

b Length of minor arc ADB $= \frac{\theta}{360^0} \times 2\Pi r$

$= \frac{\cancel{120}^{1}}{\cancel{360}_{3}} \times 2 \times \frac{22}{7} \times \cancel{6}^{2}cm$

$= \frac{4 \times 22}{7}cm$

$= \frac{88}{7}cm = 12\frac{4}{7}cm$

c. Perimeter of minor segment ACBD

= Length of + length of arc

Chord AB ADB

$= 2r \text{ Sin } \theta + 12\frac{4}{7}cm$

$= 2 \times 6 \times \sin 60^0 + 12\frac{4}{7}cm$

$= 12 \times \text{Sin } 60^0 + 12\frac{4}{7}cm$

$= 12 \times 0.8660 + 12.5714cm$

$= 10.3920 + 12.5714cm$

$= 10.3920$

$\underline{12.5714}$

$\underline{22.9634cm}$

$= 22.96$ cm to 2 places of decimal.

EVALUATION

1. a. A chord 4.8cm long is drawn in a circle of radius 2.6cm. Calculate the distance of the chord from the centre of the circle.
   b. Calculate the angle subtended at the centre of the circle by the chord in Question 1(a) above
   c. Hence find the perimeter of the minor segment formed by the chord and the minor arc of the circle.

READING ASSIGNMENT

NGM SS BK 2, pg. 31,Ex2a, Nos.2,3,5.

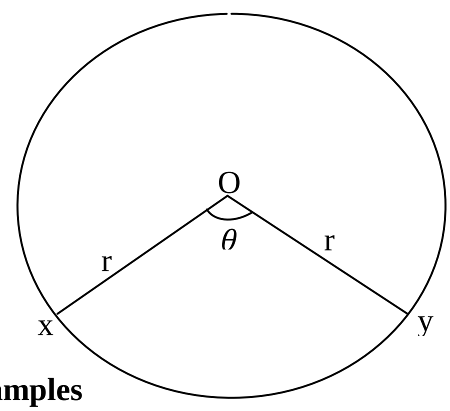
(c) Area of sectors and segments. Area of triangles

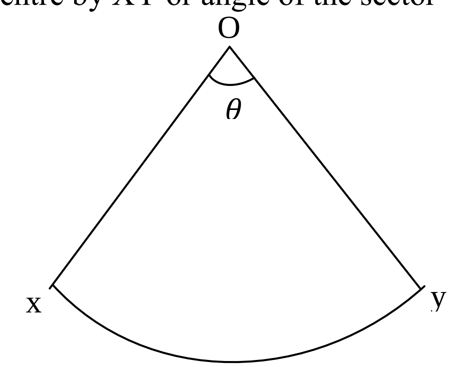
Area of sectors

Area of a sector of a circle is given by the formular;

Area of sector $\frac{\theta}{360^o} \times \pi r^2$

where r = radius of the circle, θ = angle subtended at the centre by XY or angle of the sector





Examples

1. Calcualte the area of sector of a circle which subtends an angle of 45^o at the centre of the circle, diameter 28cm ($\pi = 22/7$).
2. The area of a circle PQR with centre O is $72cm^2$. What is the area of sector POQ, if POQ = 40^o?

Solutions

1. Since the diameter of the circle = 28cm
   d = 2r = 28
   where d = diameter and r = radius
   thus 2r = 28
   $\frac{2r}{2} = \frac{28}{2} = 14cm$

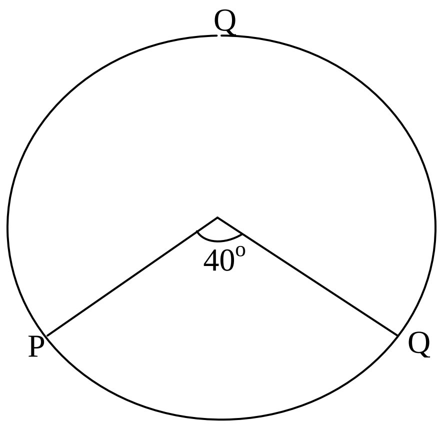
Area of sector = $\frac{\theta}{360^o} \times \pi r^2$

$= \frac{45}{360} \times \frac{22}{7} \times (14)^2$

$= 1/8 \times 22/7 \times 14 \times 14$ cm

$= 77cm^2$

2.



Since the area of the whole circle PQR = $72cm^2$

Then

Area of sector = $\frac{\theta}{360^o} \times \pi r^2$

But πr^2 = Area of the whole circle PQR = $72cm^2$

:. Area of sector POQ = $\frac{40}{360^o} \times 72cm^2$

= $8cm^2$

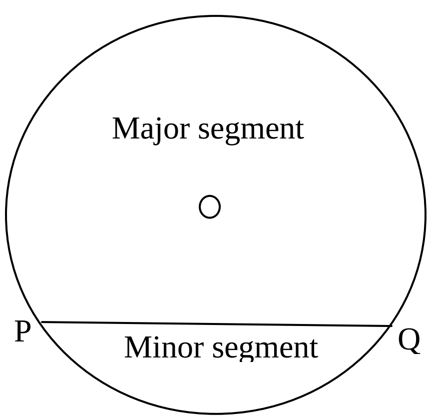
Evaluation

complete the table below for areas of sectors of circles. make a rough sketch in each case.

Radius	Angle of sector	Area of sector
a.14cm	-	$462cm^2$
b. --	140	$99cm^2$

Area of segments

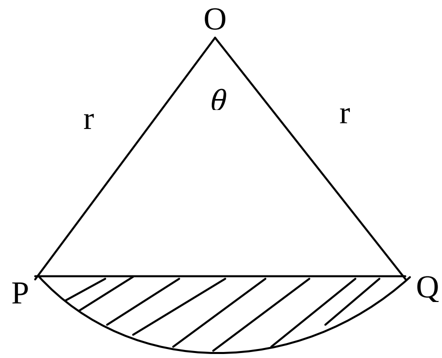
A segments of a circle is the area bounded by a chord and an arc of the circle.Considering the figure below, we have a major segment and a minor segment .



Given the diagram below:

Area of the shaded segment= Area of sector POQ – Area of triangle POQ

$$= \frac{\theta}{360^{o}} \times \pi r^{2} - \frac{1}{2} r^{2} \sin \theta$$



Where

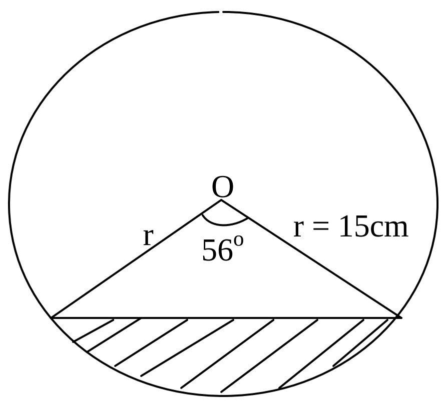
r = radius of the circle

θ = angle subtended by the sector at the centre

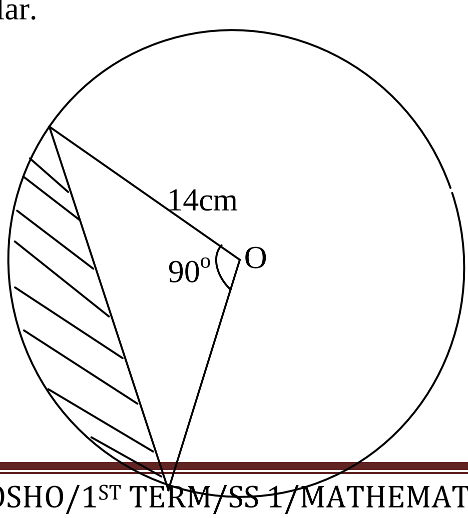
Π= a constant = 22/7

Examples

1. calculate the area of the shaded segment of the circle shown below:



2.Calculate the area of the shaded parts in the figure below. All dimensions are in cm and all arcs are circular.



Solutions

1 Area of the given shaded segment $= \frac{\theta}{360^o} \times \pi r^2 - 1/2 r^2 \sin\theta$

$= 56/360 \times 22/7 \times (15)^2 - ½ \times (15)^2 \sin 56^0$

$= 1/45 \times 22 \times 15 \times 15 - ½ \times 15 \times 15 \sin 56^o$

$= 22 \times 5 - ½ \times 225 \times \sin 56^0$

$= 110 - ½ \times 225 \times 0.8290$

$= 110 - 225 \times 0.4145$

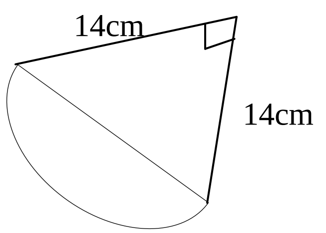
$= 110 – 93.2625 cm^2$

$= 110 – 93.2625 cm^2$

$= 16.7375 cm^2$

$= 16.7 cm^2$ to 3 s. f

2)



The arc in the given figure is part of a circle as shown in the figure above. Thus area of given shaded segment = Area of sector – area of triangle

$= \frac{\theta}{360} \times \pi r^2 – ½ r^2 \sin\theta$

$= 90/360 \times 22/7 \times (14)^2 – ½ \times (14)^2 \sin 90^o$

$= ¼ \times 22/7 \times 14 \times 14 – ½ \times 14 \times 14 \times 1$

$= 11 \times 14 – 14 \times 7 cm^2$

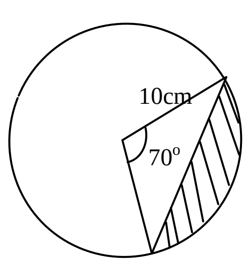
$= 154 - 98 cm^2$

$= 56 cm^2$

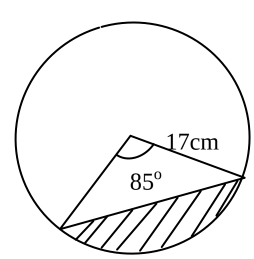
EVALUATION

Calculate the area of the shaded parts in the figure below. All dimensions are in cm and all arcs are circular.

a)

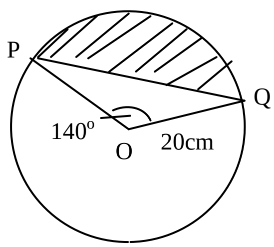


b)



GENERAL EVALUATION

1. An arc of a circle radius 7cm is 14cm long. What angle does the arc subtend at the centre of the circle?
2. An arc of a circle whose radius is 10cm subtends an angle 60^0 at the centre. Find the length of the arc.
3. In the diagram below, O is the centre of the circle of radius 20cm. Calculate:
   (a) The area of the minor segment PQ
   (b) The area of the major segment PQ
   (c) The perimeter of the minor segment. (take π = 3.13)

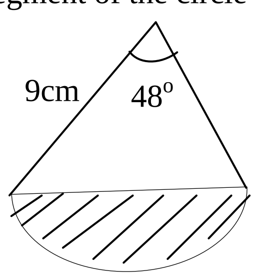


READING ASSIGNMENT

NGM SS BK1 Pages 134-139 Ex 12d Nos 6 and 9 139

WEEKEND ASSIGNMENT

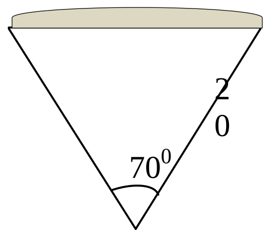
1. Calculate the area of a sector of a circle of radius 6cm which subtends an angle of 70^o at the centre (π = 22/7) A. $44cm^2$ B. $22cm^2$ C. $66cm^2$ D. $11cm^2$ E. $16.5cm^2$
2. What is the angle subtended at the centre of a sector of a circle of radius 2cm if the area of the sector is $2.2cm^2$? (π = 22/7)A. 120^o B. 31 ½^o C. 43^o D. 58^o E. 63^o
3. What is the radius of a sector of a circle which subtends 140^o at its centre and has an area of $99m^2$?
   A. 18m 27m C 9m E. 30m E. 24m
4. A sector of 80^o is removed from a circle of radius 12cm What area of the circle is left? A. $253cm^2$ B. $704cm^2$C $176cm^2$D. $125cm^2$ E. $352cm^2\pi$
5. Calculate the area of the shaded segment of the circle shown in the figure below:
   (π = 22/7)



A. $10.45cm^2$ B. $20.90cm^2$C. $5.25cm^2$D. $19.0cm^2$E. $17.45cm^2$

THEORY

1. The figure below shows the cross section of a tunnel. It is in the shape of a major segment of a circle of radius 1m on a chord of length 1.6m. Calculate:
   a. the angle subtended at the centreof the circle by the major arc correct to the nearest 0.1^0
   b. the area of the cross section of the tunnel correct to 2d.p.
2. Calculate: (i) the area of the shaded segments in the following diagrams. (ii) The perimeter (Take π 3.14)



3.

	Radius	Angle at centre	Length of arc
A	21cm	________	22cm
B	____	108^o	132cm

WEEK TEN
TOPIC: LOGIC
CONTENT

- Simple true and false statements
- Negative and contra positive of simple statement.
- Antecedents, consequence and conditional statement (implication)

LOGICAL STATEMENTS
A logical statement is a declaration verbal or written that is either true or false but not both.
A true statement has a truth value T
A false statement has a truth value F
Logical statements are denoted by letters p, q, r ……
Questions, exclamations, commands and expression of feelings are not logical statements.
Ex: Which of the following are logical statements?

i. Nigeria is an African country (Statement)
ii. Who is he? (Not statement)
iii. If I run I shall not be late (Statement)
iv. Japanese are hardworking people (Statement)
v. What a lovely man! (Not statement)
vi. The earth is conical in shape (Statement)
vii. If I think of my family (Not statement)
viii. Take the pencil away (Not statement)

EVALUATION
State which of the statements is a logical statement

1. Caesar was great leader
2. Oh Mansa Musa, you are wonderful!
3. Is he a serious teacher at all?
4. If 6 is an odd number, then 3 + 5 = 10
5. Stop talking to the boy

6. The Broking House In Ibadan is a magnificent building

SOLUTION

1. A Logical statement
2. Not a logical statement (Exclamation)
3. Not a logical statement (Question)
4. A logical statement
5. Not a logical statement (command)
6. A logical statement

   Reading Assignment: Further Maths Project Ex 9a Q 1&2

NEGATION

Given a statement p, the negation of p written ~p is the statement 'it is false that p" or "not p"

If P is true,$^{(T)}$ ~p is false$^{(F)}$and if P is false$^{(F)}$~p is true$^{(T)}$.

The relationship between P and ~p is shown in a table called a truth table

	P	~p
	T	F
F	T	

Ex I: Let P be the statement 'Nigeria is a rich country' then ~p is the statement 'It is false that Nigeria is a rich country or 'Nigeria is not a rich country'

Ex II: Let r be the statement $3 + 4 = 8$ then ~p is the statement $3 + 4 \neq 8$

Ex III: Let q be the statement 'isosceles triangle are equiangular' then ~q is the statement 'it is false that isosceles triangles are equiangular or 'isosceles triangle are not equiangular'.

EVALUATION

1. Write the negation each of the following statements.
   1. It is very hot in the tropics.
   2. He is a handsome man.
   3. The football captain scored the first goal.
   4. Short cuts are dangerous.
2. Write the negation of each of the following avoiding the word 'not' as much as possible.
   1. He was present in school yesterday.
   2. His friend is younger than my brother.
   3. She is the shortest girl in the class.
   4. He obtained the least mark in the examination.

READING ASSIGNMENT

Further maths projects Ex. 9a Q 3 – 7.

CONDITIONAL STATEMENTS

Let q stand for the statement 'Femi is a brilliant student' and r stand for the statement 'Femi passed the test'. One way of combining the two statements is 'If Femi is a brilliant student then Femi passed the test' or 'If q then r'

The statement 'If q then r' is a combination of two simple statements q and r. It is called a compound statement.

Symbolically, the compound statement can be written as follows: 'If q then r' as $q \Rightarrow r$

The statement $q \Rightarrow r$ is real as
q implies r or
if q then r or
q if r

The symbol $\Rightarrow$ is an operation. In the compound statement $q \Rightarrow r$, the statement q is called the antecedent while the sub statement r is called the consequence of $q \Rightarrow r$.

The truth or falsity table for $q \Rightarrow r$ is shown below.

q	r	$q \Rightarrow r$
T	TT	
T	F	F
F	T	T
F	F	T

Ex: If q is the statement 'I am a male' and r is the statement 'The sun will rise'
Consider the statements.

a. If I am a male then the sun will rise
b. If I am a male then the sun will not rise
c. If I am not a male then the sun will rise
d. If I am not a male then the sun will not rise

The statement (a), (c) and (d) are all true but b is not true because the antecedent is true and the consequent is false.

CONVERSE STATEMENT: The statement $q \Rightarrow p$ is called the converse of the statement $p\Rightarrow q$. e.g. Let p be the statement 'a triangle is equiangular' and q the statement 'a triangle is equilateral'.

The State $p \Rightarrow q$ means if a triangle is equiangular then it is equilateral.

The statement $q \Rightarrow p$ means if a triangle is equilateral then it is equiangular.

INVERSE STATEMENT: This statement $\sim p \Rightarrow \sim q$ is called the inverse of the statement $p \Rightarrow q$. If p is the statement 'a triangle is equiangular and q is the statement 'a triangle is equilateral' the statement$\sim p \Rightarrow \sim q$ is the statement 'if a triangle is not equiangular then it is not equilateral'.

CONTRAPOSITIVE STATEMENT: The statement $\sim q \Rightarrow \sim p$ is called the contrapositive statement of $p \Rightarrow q$.

If p is the statement 'I can swim' and q is the statement 'I will win' then the statement $\sim q \Rightarrow \sim p$ is the statement 'If I cannot swim then I cannot win'.

EVALUATION

If p is the statement 'it rains sufficiently' and q the statement 'the harvest will be good' write the symbol of these statements.

(i) If it rains sufficiently then the harvest will be good.
(ii) If it doesn't rain sufficiently then the harvest will be poor.
(iii) If the harvest is poor then it doesn't rain sufficiently.
(iv) It doesn't rain sufficiently.
(v) If it doesn't rain sufficiently then the harvest will be good.

IDENTIFICATION OF ANTICEDENCE AND CONSEQUENCE OF SIMPLE STATEMENTS.

1. Biconditional statements
2. The Chain Rule

1. **BICONDITIONAL STATEMENTS :** If p and q are statements such that $p \Rightarrow q$ and $q \Rightarrow p$ are valid, then p and q imply each other or p is equivalent to q and we write $p \Leftrightarrow q$. The statement $p \Leftrightarrow q$ is called a biconditional statement of p and q and the statement p and q are equivalent to each other.

$p \Leftrightarrow q$ could be read as
q is equivalent to p or
q if and only if p or
p if and only if q or
if p then q and if q then p

The truth or falsity of $p \Leftrightarrow q$ is shown below.

P	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

A biconditional statement is true when two sub-statements have the same truth value.

e.g. If p is the statement 'the interior angle of a polygon are equal' and q is the statement 'a polygon is regular'.

$p \Rightarrow q$ is the statement 'if the interior angles of a polygon are equal then the polygon is regular'.
$q \Rightarrow p$ is the statement 'if a polygon is regular then the interior angles of the polygon are equal'.
$p \Rightarrow q$ and $q \Rightarrow p$
$p \Leftrightarrow q$
p and q are equivalent to each other.

Examples: Let p be the statement 'Mary is a model'
Let q be the statement 'Mary is beautiful'
Consider these statements.

a. Mary is a model if and only if she is beautiful.
b. Mary is a model if and only if she is ugly.
c. Mary is not a model if and only if she is beautiful.

d. Mary is not a model if and only if she is ugly.

Statements a and d are true because the sub-statements have the same truth value. Statements b and c are false because the sub-statements have different truth values.

2. THE CHAIN RULE : If p, q and r are three statements such that $p \Rightarrow q$ and $q \Rightarrow r$.

Ex I: Consider the arguments

Premise T_1: If a student works very hard, he passes his examination

Premise T_2: If a student passes his examination he is awarded a certificate.

Conclusion T_3: If a student works very hard, he is awarded a certificate.

SOLUTION

Let p be the statement "a student works very hard"

Let q be the statement "a student passes his examination"

Let r be the statement "a student is awarded a certificate"

'The argument has the following structural form.

$p \Rightarrow q$ and $q \Rightarrow r \ \therefore \ p \Rightarrow r$

This argument follows the chain rule link hence it is said to be valid.

Ex II: Consider the arguments

T_1: Soldiers are disciplined

T_2: Good leaders are disciplined men

T_3: Soldiers are good leaders.

SOLUTION

Let p be the statement 'X is a seller'

Let q be the statement 'X is a disciplined man'

Let r be the statement 'X is a good leader'

The argument has the following structural form.

$T_1 : p \Rightarrow q$

$T_2 : r \Rightarrow q$

$T_3 : p \Rightarrow r$

The argument does not follow the format of the chain rule, hence it is not valid.

EVALUATION

Give an outline of the structural form of the following arguments and state whether or notit is valid.

T_1 : It is necessary to stay healthy in order to live long.

T_2 : It is necessary to eat balanced diet in order to stay healthy.

T_3 : It is necessary to eat balanced diet in order to live long.

GENERAL EVALUATION

1. **Determine which of the following are true and which are false.**
   (a) $(5 = 8 - 2) \wedge (4 + 7 = 11)$
   (b) $(15 > 10) \wedge (0 > - 12)$
   (c) (3, 4, 5) is a Pythagorean triples or (9, 12, 15) is a Pythagorean triples.
2. Write the converse and the inverse of the following implications:
   (a) If the bus has a driver, then the bus can carry the passengers.

(b) $M \Longrightarrow N$
(c) $A \Longrightarrow \sim B$

READING ASSIGNMENT
WABP Essential Mathematics page 189 – 190 exercise 14.3 no 5 – 10

WEEKEND ASSIGNMENT
P is the statement 'Ayo has determination and q is the statement 'Ayo will succed'. Use this information to answer these questions.
Which of these symbols represent these statements?

1. Ayo has no determination.
A. $P \Rightarrow q$ B. $\sim p \Rightarrow q$ C. $\sim p$
2. If Ayo has no determination then he won't succeed.
A. $\sim p \Rightarrow \sim q$ B. $p \Rightarrow \sim q$ C. $p \Rightarrow q$ D. $p \Rightarrow \sim q$
3. If Ayo won't succeed then he has no determination.
A. $\sim q \Rightarrow p$ B. $\sim q \Rightarrow \sim q$ C. $\sim q \Rightarrow p$ D. $q \Rightarrow p$
4. If Ayo has determination then he will succeed.
A. $\sim p \Rightarrow q$ B. $\sim p \Rightarrow \sim q$ C. $\sim q \Rightarrow \sim p$ D. $p \Rightarrow q$
5. If Ayo has no determination then he will succeed.
A. $\sim p \Rightarrow q$ B. $\sim q \Rightarrow \sim p$ C. $\sim p$ D. $\sim p \Rightarrow \sim q$

THEORY
1. Write down the inverse, converse and contrapositive of each of these statements.
(i) If the bank workers work hard they will be adequately compensated.
(ii) If he is humble and prayerful, he will meet with God's favour.
(iii)If he sets a good example, he will get a good followership.
2. Find the truth value of these statements.
a. If $11 > 8$ then $-1 < -8$
b. If $3 + 4 \neq 10$ then $2 + 3 \neq 5$