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SECOND TERM E-LEARNING NOTE

SUBJECT: FURTHER MATHEMATICS **CLASS: SS2**

SCHEME OF WORK

WEEK	TOPIC
1.	Differentiation: Limits of Function and First Principle, Differentiation of Polynomial
2.	Differentiation (Continued): Rules of Differentiation
3.	Differentiation of Transcendents: Derivative of Trigonometric Functions and Exponential Functions.
4.	Application of Differentiation: Rate of Change, Equation of Motion, Maximum and Minimum Points and Values of Functions.
5.	Conic Sections: Equation of Circles, General Equation of Circles, Finding Centre and Radius, Equation and Length of Tangents to a Circle.
6.	Conic Sections: The Parabola, Hyperbola and Ellipse
7.	Review of First Half Term
8.	Statistics Probability: Sample Space, Event Space, Combination of Events, Independents and Dependent Events.
9.	Permutation and Combination
10.	Dynamics: Newton's Laws of Motion
11.	Work, Energy, Power, Impulse and Momentum
12.	Revision and Examination.

REFERENCES

Further Mathematics Project 2 and 3.

WEEK ONE

TOPIC : LIMITS OF FUNCTIONS AND DIFFERENTIATION FROM THE FIRST PRINCIPLE

The followings are the properties of limits:

(i) lim k = k i e
$x^2 \to a$
The limit of a constant is the constant itself

(ii) lim [f(x) + f_2 (x) + $f_{3\,(x)}$ + … f_n(x)]
= lim f_1(x) + lim f_2 (x) + lim f_3 (x) +limf_n(x)
$x \to a$ $x \to a$ $x \to a$ $x \to a$
i.e
The limit of the sum of a finite number of functions is equal to the sum of their respective limits
lim [f_1(x) – f_2(x)] = limf_1(x) – limf_2(x)
$x \to a$ $x \to a$ $x \to a$ $x \to a$

i.e

The limit of the difference of two functions is equal to the difference of their limits.

(iii) $\lim_{x \to a} [f_1(x) f_2(x) f_3(x) + \ldots f_n(x)]$

$= \lim_{x \to a} f_1(x) \lim_{x \to a} f_2(x) \lim_{x \to a} f_3(x) \lim_{x \to a} f(x)$

i.e

The limit of the product of infinite number of functions is equal to the product of their respective limits.

(iv) $\lim_{x \to a}\left[\frac{f(x)}{f(x)}\right] = \frac{\lim f_1(x)}{\lim f_2(x)}$

Provided $\lim f_2(x) \neq 0$ i.e

The limit of the quotient function is equal to the quotient of their limits provided the limit of the divisor is not equal to zero

(v) $\lim_{x \to a} k f(x) = k \lim_{x \to a} f(x)$

i.e

Limit of the product of a constant and a function is equal to the product of the constant and the limit of the function

Example 1

Evaluate $\lim(7 - 2x + 5x^2 - 4x^3)$

Solution

$\lim_{x \to a} \{7 - 2x + 5x^2 - 4x^3)$

$= \lim_{x \to a} 7 - 2 \lim_{x \to a} x + 5 \lim_{x \to a} x^2 - 4 \lim_{x \to a} x$

$= 7 - 0 + 0 = 7$

Example 2

$\lim_{x \to 0} \frac{x^2 + 5x + 9}{2x^2 - 3x + 15}$

Solution

$\lim_{x \to 0} \frac{x^2 + 5x + 9}{2x^2 - 3x + 15} = \frac{\lim x^2 + 5x + 9}{\lim 2x^2 - 3x + 15}$

$\frac{\lim_{x \to 0} x^2 + 5\lim_{x \to 0} x + \lim_{x \to 0} 9}{2 \lim_{x \to 0} x^2 - 3\lim_{x \to 0} x + \lim_{x \to 0} 15}$

$= \frac{0+0+9}{0-0+15}$

$= \frac{9}{15}$

$= \frac{3}{5}$

Example

Evaluate $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

Solution

Lim $x^2 - 25$ = lim$\frac{(x+5)(x-5)}{x-5}$ =
x→ 5x – 5
= lim (x + 5)
x – 5
= lim x + lim 5
x – 5x – 5
= 5 + 5
= 10

Example
Evaluate lim $\frac{3x^3+2x^2+x+1}{x^3+2x+5}$
x – 5
Solution
We know that lim$\frac{1}{x}$ = 0
x – 0
lim$\frac{3x^3 + 2x^2 + x + 1}{x^3 + 2x + 5}$
x – 0
$x^3(3 + \frac{2}{x} + \frac{1}{2} + \frac{1}{3})$
limx x= 0
x→0 $x^3 (\frac{2}{2} + \frac{5}{3})$
x x

lim 3 + 2 lim$\frac{1}{x}$ + lim $\frac{1}{2}$ + lim $\frac{3}{3}$
=x – 0x – 0x – 0x – 0
lim1 + 2 lim$\frac{1}{x}$+ lim$\frac{1}{2}$ + 5 lim
x – 0x – 0x – 0x – 0
= $\frac{3+0+0+0}{1+0+0}$
= $\frac{3}{1}$
= 3

EVALUATION
Evaluate lim -> 4 x^3 +4x 6
Evaluastelim x -> -2 x+6/ 2x +4

Differentiation From first Principle
The technique adopted in unit 11.3 in finding the derivative of a function from the consideration of the limiting value is called **differentiation from first principle.**

Example
Find the derivative of f(x) = x^2 from first principle.
Solution
$f(x) = x^2$
$f(x + \Delta x) = (x + \Delta x)^2$
$= x^2 + 2x\Delta x + (\Delta x)^2$
$f(x + \Delta x) - f(x) = (x + \Delta x)^2 - x^2$
$= x^2 + 2x\Delta x + (\Delta x)^2 - x^2$

$= 2x\Delta x + (\Delta x)^2$

$\frac{f(x+\Delta x)-f(x)}{\Delta x} = 2x + \Delta x$

$\lim \frac{f(x+\Delta x)-f(x)}{\Delta x} = 2x$

$\Delta \to 0$

$\therefore f^1(x) = 2x$

Example

Find the derivative of $y = x^3$ from first principle

Solution

$y = x^3$

$y + \Delta y = (x + \Delta x)^3$

$= x^3 + 3x^3\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$

$\Delta y = (x + \Delta x)^3 - x^3$

$= 3x^2\Delta + 3x(\Delta x)^2 + (\Delta x)^3$

$\therefore \frac{\Delta y}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2$

$\lim \frac{\Delta y}{\Delta x} = 3x^2$

$x \to 0$

Hence $\frac{dy}{dx} = 3x^2$

Example

Find the derivative of $y = \frac{1}{x}$ from first principle.

Solution

$y = \frac{1}{x}$

$y + \Delta y = \frac{1}{x+\Delta x} - \frac{1}{x}$

$\Delta y = \frac{x-(x+\Delta x}{(x+\Delta x)x}$

$= \frac{x-x-\Delta x}{(x+\Delta x)x}$

$= \frac{-\Delta x}{x(x+\Delta x)}$

$= \therefore \frac{\Delta □}{\Delta □} = \frac{-\Delta □}{□(□+\Delta □)\Delta □}$

$= \frac{-1}{□(□+\Delta □)}$

$\lim \frac{\Delta □}{\Delta □} = -\frac{1}{□2}$

$= \Delta □ \to 0$

Hence $\frac{□□}{□□} = -\frac{1}{□2}$

Example

Find the derivative of y = c, where c is a constant, from first principle.

Solution

$y = c$

$y + \Delta □ = c$ (since c is a constant)

$\Delta □ = 0$

$\frac{\Delta □}{\Delta □} = 0$

$\lim \frac{\Delta □}{\Delta □} = 0$

$= \therefore \frac{□□}{□□} = 0$

Hence the **derivative of a constant is zero.**

EVALUATION

Find the derivative of the following using first principle.

1. $y=3x^2 + 4$ (2) $y= x^3 -2x^2 + 2x -5$

GENERAL EVALUATION

1) Evaluate (i) lim x-> 0 $x^4 + 5x / x^2 + 3$ (ii) lim x-> 2 3x + 7

2) Differentiate from the first principle $y= 2x^2 +3x + 5$

3) Find the gradient function of $y = x^2 +3x +1$ (4) Differentiate $y =5x^4 +7x^3 + 6x^2 – 9x +4$

READING ASSIGNMENT: New further Maths Project 2 page 113- 120

WEEKEND ASSIGNMENT

1) Evaluate $\lim_{x\to 1} 4x^2 + 3x$ a) 4 b) 3 c) 7 d) 0

2) Evaluate $\lim_{x\to 0} x^2 + 9$ a) 3 b) 9 c) 6 d) 1

3) Evaluate $\lim_{x\to 0}$ (x+3) (3x-3) a) 27 b) 6 c) 9 d) -9

4) Differentiate $8x^2 + 10$ a) 8x b) 16x c) 10 d) 18x

5) Find the derivative of y = b where b is a constant a) 0 b) bx c) x d) 1

THEORY

1) Evaluate $\lim_{x\to -2} 3x^3 +4 / x^2 +4$ (2) Differentiate from the first principle $y = 7x^3 + 5x^2 – 6x +5$

WEEK TWO

TOPIC: RULES OF DIFFERENTIATION

Derivative of Sum

Let *f, U* and *V* be functions of x such that

$f(x) = U(x) + V(x)$

$f(x + \Delta x) = U(x + \Delta x) + V(x + \Delta x)$

Therefore $f(x + \Delta x) – f(x) = \{U(x + \Delta x) + V(x + \Delta x) - U(x) – V(x)\}$

$= U(x) + \Delta x) – U(x) + V(x + \Delta x) – V(x)$

Therefore $\frac{f(x + \Delta x) – f(x)}{\Delta x} = \frac{U(x + \Delta x) – U(x)}{\Delta x} + \frac{V(x + \Delta x) – V(x)}{\Delta x}$

$\text{Lim} \frac{f(x + \Delta x) – f(x)}{\Delta x} = \text{Lim} \frac{U(x + \Delta x) – U(x)}{\Delta x} + \text{Lim} \frac{V(x + \Delta x) – V(x)}{\Delta x}$

Therefore $f'(x) = U'(x) + V'(x)$

In other words, if y = U + V, where U and V are functions of x, then:

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Hence, the derivation of a sum is the sum of the derivatives.

Examples

Find the derivative of each of the following

(a) $2x^3 – 5x^2 + 2$

(b) $3x^2 + \frac{1}{x}$

(c) $x^3 + 2x^2 + 1$

$\overline{\quad\quad\quad}$
x

(d) $\sqrt{x}$ + $1\sqrt{x}$ - 3

Solution

(a) Let y = $2x^3 - 5x^2 + 2$

$\frac{dy}{dx}$ = $6x^2 - 10x$

(b) Let y = $3x^2 + \frac{1}{x}$

= $3x^2 + x^{-1}$

$\frac{dy}{dx}$= 6x - $\frac{1}{x2}$

(c) let y = $\frac{x^3 + 2x^2 + 1}{x}$

= $x^2 + 2x + x^{-1}$

$\frac{dy}{dx}$ = 2x + 2 - $\frac{1}{x2}$

(d) y = $\sqrt{x}$ + $1\sqrt{x}$ - 3

= $x^{\frac{1}{2}}$ + $x^{\frac{-1}{2}}$ -3

$\frac{dy}{dx}$ = $\frac{1}{2}$ x $\frac{-1}{2}$ - $\frac{1}{2}$ x $\frac{-3}{2}$

= 1 ÷ $\sqrt[2]{x}$ - 1 $\sqrt[2]{x3}$

Functions of a Function

Suppose we know that y is a function of u and the u itself is also a functions of x, how do we find the derivation of y with respect to x?

In other words, given y = f (u) and u = h(x)

What is $\frac{dy}{dx}$? By simple re – arrangement we can write

$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \times \frac{\Delta u}{\Delta x}$; $\Delta u \neq 0$, $\Delta x \neq 0$

Lim $\frac{\Delta y}{\Delta x}$ = Lim $\left\{\frac{\Delta y}{\Delta u} \times \frac{\Delta u}{\Delta x}\right\}$

= $\lim \frac{\Delta y}{\Delta u}$ x $\lim \frac{\Delta u}{\Delta x}$

As $\Delta x \rightarrow 0$, $\Delta u \rightarrow 0$

So we can

$\frac{\lim}{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ as $\lim \frac{\Delta y}{\Delta u}$

Thus

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \times \lim_{\Delta x \to 0}$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

This is called the chain rule for differentiation.

Examples

Find the derivative of each of the following:

(a) $y = (3x^2 - 2)^3$

(b) y =

(c) $y = \sqrt{(1-2x^3)}$

(d) $y = \overline{(6-x^2)}^3$

(e) $\frac{1}{\sqrt{(1+x^2)^3}}$

solution

(a) Given $y = (3x^2 - 2)^3$

Let $u = 3x^2 - 2$

then $y = u^3$

$\therefore \frac{dy}{du} = 6x$

$\frac{dy}{dx} = \frac{dy}{du} \quad \frac{dy}{dx}$

$= 3u^2 \times 6x$

$= 18xu2$

$= 18x(3x^2 - 2)^2$

(b) $y = \sqrt{(1-2x^3)}$

Let $u = 1-2x^3$

then $y = u^{\frac{1}{2}}$

$\frac{dy}{du} = \frac{1}{2} u^{-½}$

$= \frac{1}{\sqrt[2]{u}}$

$= -6x^2$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{\sqrt[2]{u}} \times -6x^2$

$\frac{dy}{dx} = \frac{-3x^2}{\sqrt{(1-2x^3)}}$

(c) $y = \frac{1}{(6-x^2)^3}$

Let $u = 6 - x^2$

then $y = \frac{5}{u^3}$

$= 5u^{-3}$

$\therefore \frac{dy}{du} = -15u^{-4}$

$\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -15u^{-4} \times -2x$

$= 30 \times u^{-4}$

$\frac{dy}{dx} = (6-x^2)^4$

(d) $y = \frac{5}{\sqrt{(1+x^2)}}$

Let $u = 1 + x^2$

then $y = \_\_\_ = 1$

$= u^{\frac{1}{2}}$

$\frac{dy}{du} = -\frac{1}{2} u^{\frac{-3}{2}}$

$= \frac{1}{-2\sqrt{u3}}$

$\frac{du}{dx}$= 2x

$\frac{dy}{dx}$ = $\frac{dy}{du}$ x $\frac{du}{dx}$

-1

= $2\sqrt{u3}$ x 2x

= $\frac{-x}{\sqrt{(1+x2)3}}$

EVALUATION

Find the derivative of the followings:

(i) $y = 8x^5 + 6x - 7$

(ii) $y = (4x^3 - 3)^4$

(iii) $y = 3x^2 + 1/x^3 + 2/x$

The Derivation of a Product

We shall now consider the derivative of y = uv where u and v are functions of x.

Let y = uv

Then $y + \Delta y = (u + \Delta u)(v + \Delta v)$

$= uv + u\Delta v + v\Delta u + \Delta u\Delta v - uv$

$= u\Delta v \;\; v\Delta u + \Delta u \, \Delta v$

$\frac{\Delta y}{\Delta x} = u\frac{\Delta v}{\Delta x} + v\frac{\Delta u}{\Delta x} + \Delta u \frac{\Delta}{\Delta x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Examples

Find the derivative of each of the following

(a) $y = (3 + 2x)(1 - x)$

(b) $y = (1 - 2x + 3x^2)(4 - 5x^2)$

(c) $y = \sqrt{x}(1 + 2x)^2$

(d) $y = x^3(3 - 2x + 4x^2)^{1/2}$

Solution

(a) $y = (3 + 2x)(1 - x)$

Let u = 3 + 2x; v = 1 – x

$\frac{du}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

= (3 + 2x) x – 1 + (1 – x) x 2

= -(3 + 2x) + 2(1 – x)

= -3 -2x + 2 – 2x

= - 1 – 4x

(b) $y = (1 - 2x + 3x^2)(4 - 5x^2)$

Let u + 1 – 2x + 3x2; v =4 – 5 x^2

$\frac{du}{dx}$ = -2 + 6x; $\frac{dv}{dx}$= -10x

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

= (1 -2x + $3x^{2)}$ x (-10x)

+ $(4 - 5x^2)$

(c) y = $\sqrt{x}(1 + 2x)^2$

Let u = $\sqrt{x}$; v = $(1 + 2x)^2$

1

$\frac{du}{dx} = \sqrt[2]{x}$; $\frac{dv}{dx} = 4(1 + 2x)$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= \sqrt{x}\, 4(1 + 2x) + (1 + 2x)^2 \times \frac{1}{\sqrt[2]{x}}$

$= 4\sqrt{x}\,(1 + 2x) + \frac{\frac{1}{2}(1 + 2x)^2}{\sqrt{x}}$

(d) $y = x^3 (3 – 2x + 4x^2)^{1/2}$

Let $u = x^3$; $v = (3 – 2x + 4x^2)^{1/2}$

$\frac{du}{dx} = 3x^2$; $\frac{dv}{dx} = \frac{1}{2}(-2 + 8x) \times (3 – 2x + 4x^2)^{-1/2}$

$\frac{du}{dx} = 3x^2$; $\frac{dv}{dx} = \frac{4x-1}{(3-2x+4x2)}{}^{1/2}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= \frac{x^3 (4x – 1)}{(3x – 2x + 4x^{2)})^{½}} + 3x^2 \times (3 – 2x + 4x^{2)})^{½}$

$= \frac{x^3 (4x -1) + 3x^2 (3 -=2x + 4x^{2})}{(3x – 2x + 4x^2)^{½}}$

The Derivative of a Quotient

Let $y = \frac{u}{v}$, where u and v are functions of x and v ≠ 0.

$y + \Delta y = \frac{u + \Delta u}{v + \Delta v}$

$\Delta y = \frac{dy}{dx} = v\frac{du}{dx} - u\frac{dv}{dx}$

Examples

Find the derivative of each of the following:

(a) $\frac{1+x2}{1-x2}$

(b) $\frac{3 + 2x – x^2}{\sqrt{1 + x}}$

(c) $\frac{2 + x}{x^2 + 2x + 7}$

(d) $\frac{3\sqrt{(1+3x2)2}}{x}$

Solution

(a) $y = \frac{1 + x^2}{1 – x^2}$

Let $u = 1 + x^2$; $v = 1 – x^2$

$\frac{du}{dx} = 2x$; $\frac{dv}{dx} = -2x$

$\frac{dy}{dx} = v\frac{du}{dx} - u\frac{dv}{dx}$

$= \frac{(1 – x^2) \times (2x) – (1 + x^2) \times (- 2x)}{(1 – x^2)^2}$

$= \frac{2x – 2x^3 + 2x + 2x^3}{(1 – x^2)}$

$= \frac{4x}{(1 – x^2)^2}$

(b) $y = \frac{3 + 2x - x^2}{\sqrt{1+x}}$

Let $u = 3 + 2x - x^2$; $= (1 - x)^{\frac{1}{2}}$

$\frac{du}{dx} = 2 - 2x$; $\frac{dv}{dx} = \frac{1}{2}(1 + x)^{\frac{-1}{2}}$

$\frac{dy}{dx} = \frac{(1 + x)^{\frac{1}{2}} \text{ x } 2(1 - x) - (3 + 2x - x^2) \text{ x } \frac{1}{2(1+x)}}{1 + x}$

$= \frac{4(1 + x)(1 - x) - (3 + 2x - x^2)}{2(1 + x)(1 + x)^{\frac{1}{2}}}$

$= \frac{4 - 4x^2 - 3 - 2x + x^2}{2(1 + x)^{3/2}}$

$= \frac{1 - 2x - 3x^2}{2(1 + x)^{3/2}}$

(c) $y = \frac{2 + x}{x^2 + 2x + 7}$

Put $u = 2 + x$; $v = x^2 + 2x + 7$

$\frac{du}{dx} = 1$; $\frac{dv}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{(x^2 + 2x + 7) \text{ x } 1 - (2 + x)(2x + 2)}{(x^2 + 2x + 7)^2}$

$= \frac{x^2 + 2x + 7 - 2(x + 2)(x + 1)}{(x^2 + 2x + 7)^2}$

$= \frac{x^2 + 2x + 7 - 2x^2 - 6x - 4}{(x^2 + 2x + 7)^2}$

$= \frac{x^2 + 2x + 7 - 2x^2 - 6x - 4}{(x^2 + 2x + 7)^2}$

$\frac{dy}{dx} = \frac{- x^2 - 4x + 3}{(x^2 + 2x + 7)^2}$

HIGHER DERIVATIVES OF THE SECOND AND THIRD ORDER .
DIFFERENTIATION OF IMPLICIT FUNCTIONS

HIGHER DERIVATIVES

Given that $y = f(x)$, $\frac{dy}{dx}$ is also a function of x.

The derivative of $\frac{dy}{dx}$ with respect to x is

$\frac{dy}{dx}\left(\frac{dy}{dx}\right)$. $\frac{dy}{dx}\left(\frac{dy}{dx}\right)$is called the second derivative of y with respect to x, and it is usually denoted

$\frac{d^2y}{dx^2}$ (read. Dee two y dee x squared).

Since $\frac{d^2y}{dx^2}$ is also a function of x, successive derivatives can be found.

The third derivative of y with respect to x

Is $\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)$ and is written for short as $\frac{d^3y}{dx^3}$

S I $\frac{d^4y}{}$ is the fourth derivative of y with respect to x.

dx^4

in general $\frac{dny}{dx}$ is the nth derivative of y with respect to x.

We recall that if y = f(x), $\frac{dy}{dx}$ is sometimes denoted f^1 (x).

Similarly $\frac{d2\,y}{dx2}, \frac{d3\,y}{dx3}, \frac{d4\,y}{dx4}$, are sometimes

Denoted $f^n(x)$, $f^m(x)$, f^{iv} (x) respectively.

Example 22

Find the first second and third derivatives of each of the following:

(a) 3x4 (b) $3x^5 - 2x^4 + x^2 - 1$

(c) Inx (d) e^{x4}

(e) $\sin 3x^2$

Solution:

(a)Let $y = 3x^4$

Then $\frac{dy}{dx} = 12x^3$

$\frac{d^2y}{dx^2} = 36x^2$

$\frac{d^3y}{dx^3} = 72x$

(b) Let $y = 3x^5 – 2x^4 + x^2 -1$

$\frac{dy}{dx} = 15x^4 – 8x^3 + 2x$

$\frac{d^2y}{dx^2} = 60x^3 - 24x^2 + 2$

$\frac{d^3y}{dx^3} = 180x^2 – 48x$

(c) Let y = Ink

$\frac{dy}{dx} = \frac{-1}{x}$

$\frac{d^2y}{dx^2} = \frac{-1}{x}$

$\frac{d^3y}{dx^3x^3} = 2$

(d) Lat y = e^{x4}

$\frac{dy}{dx} = 4x^3\ e^{x4}$

$\frac{d^2y}{dx^2} = 12x^2e^{x4} + 4^{x3}\ e^{x4}\ 4^{x3}$

$= 12x^2ex^4 + 16x^6 - e^{x4}$

$\frac{d^3y}{dx^3} = 24\ e^{x4} + 12x^2 \frac{d}{dx} (e^{x4}) + 96x^5\ e^x$

$e^{x4} + 16x^6 \frac{d}{dx} (e^{x4})$

$= 24\ e^{x4} + 48x^5\ e^{x4} + 96x^5 \times e^{x4} + \qquad 64x^9\ e^{x4}$

(e) Let $y = \sin 3x^2$

$\frac{dy}{dx} = \cos 3x^2 \times \frac{dy}{dx} (3x^2)$

$= 6x\cos 3x^2$

$\frac{d^2y}{dx^2} = 6\cos 3x^2 - 6x \sin 3x^2 \frac{dy}{dx}(3x^2)$

$= 6\cos 3x^2 - 36x^2 \sin 3x^2$

$\frac{d^3y}{dx^3} = -6\sin 3x^2 \frac{d}{dx}(3x^2) - 72x$

$\times \sin 3x^2 \; 36x^2\cos 3x^2 \frac{d}{dx}(3x^2)$

$= -(6\sin 3x^2)\, 6x - 17x\sin 3x^2 - (36x^2 \cos x^2) \times 6x$

$= -36x\sin 3x^2 - 72x\sin 3x^2 - 216x^3\cos 3x^2$

$= -108x\sin 3x_2 - 216x^3 \cos 3x^2$

$= -108x(\sin 3x^2 + 2x^2\cos 3x^2)$

EVALUATION

Find the second and third derivatives of (1) cos 6x (2) $4x^5 - 5x$

Implicit Differentiation

So far, we have treated relations. Of the form y = f (x). Examples of such relations are $y = 3x^2 - 2x + 1$, $y = 1 + \sqrt{x}$

In any of these relations, y is said to be expressed explicitly in terms of x. The derivative of y with respect to x can be found from the rules of differentiation which have been discussed in the previous units.

Sometimes, the relationship between y and x may not be expressed explicitly.

For example, consider $x^2y + xy^3 + 3x = 0$. Here, the relation between y and x is not expressed explicitly. The relationship between y and x is said to be **implicit.**

In differentiating $x^2y+xy^3+3x=0$, y is treated as if it is a function of x and the rules of differentiation are applied in the appropriate manner. The process of differentiating implicit function is called **implicit differentiation.**

Examples

Differentiate each of the following implicitly:

(a) $X^2 + y^2 = 4$
(b) $X^2y + y^2 + 4x = 1$
(c) $4y^2x - 5x^2y3 + 4y = 0$
(d) $(x + y)^2 = 5$

Solution

(a) $X^2 + y^2 = 4$

Differentiating term by term with respect to x:

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$= \frac{-x}{y}$

(b) $X^2 + y^2x + 4x = 1$

$2xy + x^2\frac{dy}{dx} + 2yx\frac{dy}{dx} + y^2 + 4 = 0$

$(x^2 + 2yx)\frac{dy}{dx} = -y^2\, 2xy - 4$

$\frac{dy}{dx}$ = $\underline{-(y^2 + 2xy + 4)}$
$X^2 + 2yx$

(c) $4y^2x - 5x^2y^2 + 4y = 0$

$8yx\frac{dy}{dx} + 4y^2 - 15x^2y^2\frac{dy}{dx} - 10xy^3 + 4 \times \frac{dy}{dx} = 0$

$(8xy - 15x^2y^2 + 4)\frac{dy}{dx} = 10xy^3 - 4y^2$

$\frac{dy}{dx} = \underline{10xy^3 - 4y^2}$
$8xy - 15x^2y^2 + 4$

(d) $(x + y)^2 = 5$

$\therefore x^2 + 2xy + y^2 = 5$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$(2x + 2y)\frac{dy}{dx} = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x-2y}{2y+2y}$

$= \frac{-2(x+y)}{2(x+y)}$

$= \frac{dy}{dx} = -1$

EVALUATION

Differentiate the followings ;

(i) $y = (3x+4)(6x-8)$

(ii) $y = 6x+7/2x-3$

GENERAL EVALUATION

1) Differentiate $y = (7x^4 - 6)^5$

2) Differentiate $y = (2x + 5)(6x - 8)$

3) Find the derivative of $y = 3x^2 - 5/x + 3$

4) Find the derivative of $y = 8/(9 - x^5)^4$

5) Find the derivative of $y = 2x^4 - 5x^3 -+ 6$

6) If $x^3 - y^2 + 6xy = 0$ find dy/dx

7) Find d^3y/dx^3 given that $y = 8x^5 - 3x^4 + 9x^3 - 7x^2 + 6x + 4$

Reading Assignment

New Further MathsProject 2 page 121 – 126

WEEKEND ASSIGNMENT

1) If $y = 3x^4 - 7x + 5$ find dy/dx a) $12x^3$ b) $12x^3 - 7$ c) $12x^3 + 5$ d) $12x^3 + 12$

2) Find the second derivative of cos 5x a) 5sin5x b) -25cos5x c) 25cos5x d) -25sin5x

3) 2) If $x^2y + 4xy = 1$ find dy/dx a) $4+2xy/x^2$ b) $4-2xy/x^2$ c) $-4-2xy/x^2$ d) $-4+2xy/x$

4) Given that $y = x^2 + 3x + 2$, find dy/dx at x = 2 a) 6 b) 4 c) 7 d) 5

5) Given that $y = (2x + 3)^4$ find dy/dx a) $18(2x + 3)^3$ b) $4(2x + 3)^4$ c) $8(2x + 3)^3$ d) $2(2x+3)^3$

THEORY

1) Differentiate $y = (2x^2 - 3)^3/x$

2) Differentiate $y = (2x+ 3)^3 (4x^2 - 1)^2$

WEEK THREE

TOPIC :DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS, LOGARITHMIC FUNCTIOS AND EXPONENTIAL FUNCTIONS

DERIVATIVE OF TRIGONOMETRIC FUNCTIONS

The derivative of y = sin x dy/dx = cos x

The derivative of y = cos x dy/dx = - sin x

The derivative of y= tan x dy/dx = $\sec^2 x$\

The derivative of y = sec x dy/dx = secxtanx

The derivative of y = cosec x dy/dx = cosec x cot x

The derivative of y = cot x dy/dx = - $\text{cosec}^2 x$

Examples

(1) If y = cos 2x

dy/dx = - sin2x x d/dx (2x)

dy/dx = -2 sin 2x

(2) If y = $\cos^2 x$

Let u = cos x and y = u^2

dy/du= 2u and du/dx= -sinx

dy/dx = 2u x – sinx

dy/dx = - 2 cos x sin x

(3) If y = sec 6x

Let u = 6x and y= sec x

du/dx = 6 and dy/du = sec u tan u

dy/dx = 6 sec u tan u

dy/dx = 6 sec 6x tan 6x

EVALUATION

Differentiate the followings : (i) y = tan 8x (ii) y= cot 5x (iii) y = $\sin^4 x$

THE DERIVATIVE OF LOGARITHMIC FUNCTIONS

If y = $\log_e x$ dy/dx = 1/x**(note that $\log_e x$=lnx)**

Examples

(1) If y =$\log_e$ (3x + 2)

dy/dx =dy/du x du/dx

Let u = 3x +2 and y = $\log_e u$

du/dx = 3 and dy/du = 1/u

dy/du = 1/u x 3 = 3/3x + 2

(2) If y = $\log_e(4x – 1)^2$

Let u = $(4x – 1)^2$ and y = $\log_e u$

du/dx = du/dv x dv/dx where v = 4x – 1

du/dx =2v x 4 = 8v

dy/du = 1/u

dy/dx =dy/dx x du/dx = 1/u x 8v

dy/dx = 8 (4x – 1)/$(4x – 1)^2$

dy/dx = 8 /4x-1

EVALUATION

Differentiate the followings: (i) y = $\log_e 8x$ (ii) y = ln $(6x + 9)^3$ (iii) y = ln $(3x^2 – 5x +6)$

THE DERIVATIVE OF EXPONENTIAL FUNCTIONS

If $y = e^x dy/dx = e^x$

Examples

(1) If $y = e^{2x}$

dy/dx = dy/du x du/dx

u = 2x and $y = e^u$

du/dx = 2 and $dy/du = e^u$

$dy/dx = e^u \times 2$

$dy/dx = e^{2x} \times 2$

$dy/dx = 2e^{2x}$

(2) If $y = e^{\sin 4x}$

dy/dx = dy/du x du/dx

Let u = sin 4x and $y = e^u$

du/dx = 4 cos 4x and $dy/du = e^u$

$dy/dx = e^u \times 4\cos 4x$

$dy/dx = e^{\sin x} \times 4\cos 4x$

$dy/dx = 4e^{\sin 4x}\cos 4x$

EVALUATION

Differentiate the followings : (i) $y = e^{\tan 7x}$ (ii) $y = e^{6x}$ (iii) $y = e^{-5\sin 3x}$

GENERAL EVALUATION

(1) Find the derivative of each of the following functions : (i) $\sin^3 x$ (ii) cosec x^2

(2) Find the derivative of each of the following functions ; (i) log (x^2 -5x + 6)

(3) Differentiate each of the followings : (i) $e^{\text{cosec } x}$ (ii) $e^x - e^{-x}$

(4) Differentiate log (cos x + sin x)

Reading Assignment : New Further M aths Project 2 page 13o – 137

WEEKEND ASSIGNMENT

1) If $y = \log_e (1/x)$ find dy/dx a) 1/x b) -1/x c) $1/x^2$ d) $-1/x^2$

2) If $y = 3e^{5x}$ find dy/dx a) $3e^{5x}$ b) $15e^{3x}$ c) $15e^{5x}$ d) $5e^{5x}$

3) If y = sin 4x find dy/dx a) 4 cos 4x b) -4cos4x c) 4sin4x d) 4tan 4x

4) If y = cot 7x finddy/dx a) $7\sec^2 x$ b) $-7\text{cosec}^2 x$ c) $-7\text{cosec}^2 7x$ d) 7 tan 7x

5) Differentiate sin x – cos x a) sinx + cosx b) cosx – sinx c) sinx- cosx d) -sinx-cosx

THEORY

1) Differentiate the followings ; (i) $\cos^3 x$ (ii) sin 4x (iii) $e^{\cos 5x}$ (iv) $\cos 4x^3$

2) Differentiate the followings : (i) ln sin x (ii) log ($x^2 - 2$) (iii) log $(1 + x)^4$

WEEK 4

TOPIC: APPLICATION OF DIFFERENTIATION ; RATE OF CHANGE, EQUATION OF MOTION , MAXIMUM AND MINIMUM POINTS AND VALUES

Rate of Change

If y = *f*(x), $\frac{dy}{dx}$ can sometimes be interpreted as the rate at which y is changing with respect to x. if *y* increases as x increases, $\frac{dy}{dx} > 0$, while if *y* decreases as x increases, $\frac{dy}{dx} < 0$.

Example

The radius of a circle is increases a t the rate of 0.01cm/s. find the rate at which the area is increasing when the radius of the circle is 5cm.

Solution

Let *A*be the area of the circle of radius *r*.

$A \quad = \quad \pi r^2$

$\frac{dA}{dr} = 2\,\pi r$

By the chain rule:

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$\frac{dr}{dt} = 0.01$cm/s

When r = 5cm

$\frac{dA}{dt} = 2\pi \times 5 \times 0.01$

$= 10\pi \times 0.01$

$= 0.1\,\pi cm^2/s$

$= 0.3142 cm^2/s$

Example

Water is leaking from a hemisphere bowl of radius 20cm at the rate of 0.5cm^3/s. Find the rate at which the surface area of the water is decreasing when the water level is half-way from the top.

Solution

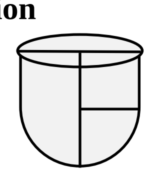


Fig. 10 9

Let *A*be the surface area of the water in the hemisphere bowl of radius *r*, then

$A = \pi r^2$

$= \frac{dA}{dr} = 2\pi r$

$= \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$V = \frac{2}{3}\pi r^3$

$\frac{dv}{dr} = 2\,\pi r^2$

$= \frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$\frac{dA}{dt} = \frac{dA}{dt}$

$\frac{dv}{dt} = \frac{dv}{dt} = \frac{2\pi r}{2\pi r2} = \frac{1}{r}$

$\therefore \frac{dA}{dt} = \frac{1}{r}\frac{dv}{dt}$

When the water level is half- way from the top r = 10cm.

But $\frac{dv}{dt}$ = -0.5 cm^3/s

$\therefore \frac{dA}{dt} = \frac{1}{10cm} \times -0.5\ cm^3/s$

= -0.5 cm^2/s

Example

Find the rate at which the volume of a spherical balloon is increasing if the surface area is increasing at the rate of $5cm^2/s$ when the radius of the spherical balloon is 4cm.

Solution

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dr} = 4\pi r^2$

$s = 4\pi r^2$

$\frac{ds}{dr} = 8\pi r$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

MAXIMUM AND MINIMUM POINTS The points on a curve at which $\frac{dy}{dx} = 0$, are called **Stationary Points**.

Stationary points fall into three major categories:

(a) Those in which $\frac{dy}{dx}$ changes sign from positives through zero to negative. These are called **maximum points.**

(b) Those in which $\frac{dy}{dx}$ changes sign from negative through zero to positive. These are called **minimum points.**

(c) Those in which the sign of $\frac{dy}{dx}$ is not changed in the immediate neighborhood of the stationary points. These are called **points of inflexion.**

The terms maximum and minimum points are used in the local sense and not in the absolute sense.

Maximum Points

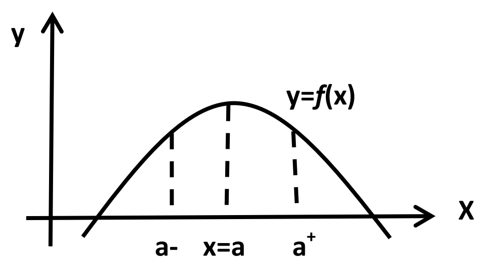


Fig.10. 11 shows part of the curve y = f (x). There is a maximum at the point where x = a $= \therefore f'(a) = 0$

Let us denote a point at the immediate neighborhood of a to the left by a and a pint at the immediate neighborhood of a to the right by a^+ then:

At $x = a$, $\frac{dy}{dx} > 0$

$\therefore f'(a^-) > 0$

At $x = a^+$, $\frac{dy}{dx} < 0$

$\therefore f'(a^+) < 0$

Hence for the existence of a minimum at x = a, three conditions must be satisfied:

(i) $f'(a) = 0$ (ii) $f'(a) > 0$

(ii) $f'(a^+) < 0$

Minimum Points

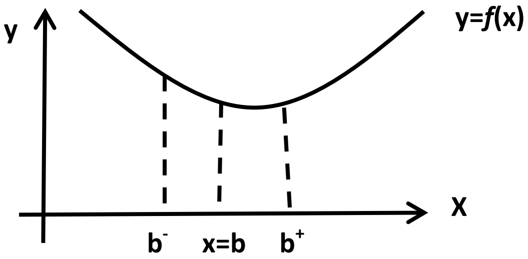


Fig. 10.12 shows a part of the curve y = *f* (x). There is a minimum at x = *b*

At x = b, f1 (b) =0

At $x = b^-$, $f^1(b^-) < 0$

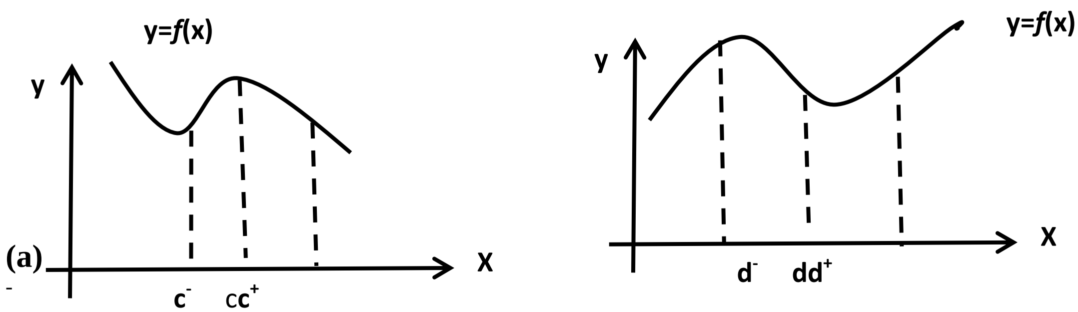
At $x = b^{+,}$ $f^1(b^+) < 0$

So a pint on a curve is a minimum at x = a.

If: (i) $f'(a) = 0$ (ii) $f'(a^-) < 0$

(iii) $f'(a^+) > 0$

Points of Inflexion



shows part of the curve y= f(x). We observe that

$f'(c) = 0$

$f'(c^-) < 0$

$f'(c^+) < 0$

The point $x = c$ is a point of inflexion. Similarly, fig. 10.13(b) shows parts of the curve y = *f* (x)

We observe that

$f'(d) = 0$

$f'(d^-) > 0$

$f'(d^+) > 0$

The point $x = d$ is a point of inflexion.

A maximum point, a minimum point and a point of inflexion are all stationary points. Both maximum and minimum points are called turning points. A point of inflexion however, is not a turning point.

Example

Find the stationary points in each of the following curves whose equations are:

(a) $y = \frac{x^3}{3} + x^2 - 3x + 4$

(b) $y = \frac{x^4}{4} + \frac{4}{3}x^3 - 2x^2 - 16x + 1$

Solution

(a) $y = \frac{x^3}{3} + x^2 - 3x + 4$

$\frac{dy}{dx} = x2 + 2x - 3$

$= (x - 1)(x + 3)$

At the stationary points, $\frac{dy}{dx} = 0$

$\therefore (x - 1)(x + 3) = 0$

$x = 1$ or $x = -3$

Hence there are stationary points at x = 1 and x = -3

$y = \frac{x^4}{4} + \frac{4}{3}x^3 - 2x^2 - 16x + 1$

$\frac{dy}{dx} = x^3 + 4x^2 - 4x - 16$

$= (x - 2)(x + 2)(x + 4)$

At the stationary points, $\frac{dy}{dx} = 0$.

$(x - 2)(x + 2)(x + 4) = 0$

$x = 2$ or $x = -2$ or $x = -4$

Hence there are stationary points at x = 2, x = -2 and x = -4

Example

Find the turning points on the curve $y = \frac{x^4}{2} + \frac{5}{3}x^3 - 2x^2 - 3x + 1$ and distinguish between them

Solution

$y = \frac{x^4}{2} + \frac{5}{3}x^3 - 2x^2 - 3x + 1$

$\frac{dy}{dx} = 2x^3 + 5x^2 - 4x - 3$

$\therefore \frac{dy}{dx} = (2x + 1)(x - 1)(x + 3)$

At the stationary points, $\frac{dy}{dx} = 0$

$(2x + 1)(x - 1)(x + 3) = 0$

$\therefore x = \frac{-1}{2}$, x = 1 and x = -3 are the x-Coordinates of the stationary points.

Let $f(x) = \frac{x^4}{2} + \frac{5}{3}x^3 - 2x^2 - 3x + 1$

$f'(x) = 2x^3 + 5x^2 - 4x - 3$

$= (2x + 1)(x - 1)(x + 3)$

Let $a = \frac{-1}{2} = -0.5$, $a^- = -5$, $a^+ = -0.4$

Then $f'(a) = 0$

$f'(a) = (-1.2+1)(-0.6 - 1)(-0.6 + 3)$

$= (-0.2)(-1.6)(2.4) > 0$

$= f^1(a) > 0$

$\therefore f'(a^+) = (-0.8 + 1)(-0.4 - 1)(-0.4 + 3)$

$= (0.2)(-1.4)(2.6) < 0$

$\therefore f'(a^+) < 0$

Table

	$f'(a^-)$	$f'(a)$	$f'(a^+)$
Sign	+ve	0	-ve
	/	---	\

Hence, there is a maximum point at $x = \frac{-1}{2}$

At $x = 1$

Let $a = 1$, $a^- = 0.9$, $a^+ = 1.1$

$f'(a^-) = 0$

$f'(a^-) = (1.8 + 1)(0.9 - 1)(0.9 + 3) < 0$

$f'(a^+) = (2.2 + 1)(1.1 - 1)(1.1 + 4) > 0$

Table

	$f'(a^-)$	$f'(a)$	$f'(a^+)$
Sign	**-ve**	**0**	**+ve**
	/	-	\

Hence, there is minimum point at $x = 1$.

At $x = -3$

Put $a = -3$, $a^- = -3.1$; $a^+ = -2.9$

$f'(a) = 0$

$f'(a^-) = (-6.2 + 1)(-3.1 - 1)(-3.1 + 3) < 0$

$f'(a^+) = (-5.8 + 1)(-2.9 - 1) 9-2.9 + 3) > 0$

Table

	$f'(a^-)$	$f'(a)$	$f'(a^+)$
Sign	-ve	0	+ve
	/	-	\

Hence, $x = -3$ is a minimum point

Example

Find the stationary points on the curve $y = x^3 - 6x^2 + 12x - 8$ and distinguish between them.

Solution

$y = x^3 - 6x^2 + 12x - 8$

$\frac{dy}{dx} = 3x^2 - 12x = 12$

$= 3(x^2 - 4x + 4)$

$= 3(x – 2)^2$

At the stationary points, $\frac{dy}{dx} = 0$

Put a = 2, $a^- = 1.9$, $a^+ = 2.1$

and let $f’(x) = x^3 – 6x^2 + 12x – 8$

$f ’(x) = 3(x^2 - 12x + 12)$

$= 3(x^{2} - 4x + 4)$

$= (3x -2)^2$

$f ’(a) = 0$

$f ’(a^-) = 3(1.9 – 2)^2 > 0$

$f ’ (a^+) = 3(2.1 – 2)^2 > 0$

Table

	f’ (a^-)	f’(a)	f’(a^+)
Sign	-ve	0	+ve
	/	-	\

Hence, there is a point of inflexion at x, =2

When x = 2

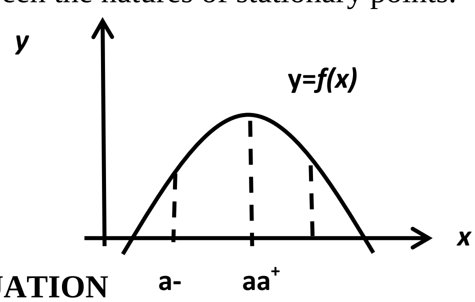
y = 8 – 24 + 24 – 8 = 0

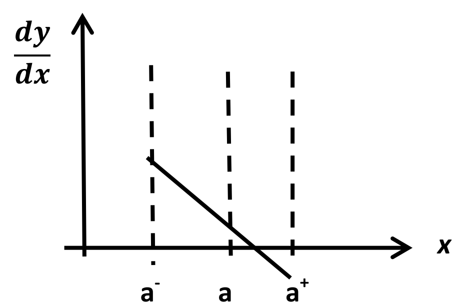
= ∴ The point (2, 0) is a point of inflexion on the curve $y = x^3 – 6x^2 + 12 – 8$

The second derivative test for stationary points

We recall that a necessary condition for the existence of stationary points for the curve. y = f (x) is that $\frac{dy}{dx}$ =. This condition is however not sufficient to determine the nature of the stationary points. We shall consider an alternative method which enables us to distinguish between the natures of stationary points.

(a)
(b)





EVALUATION

1) Find the minimum and maximum points of the curve $y= x^3 –x-5x$ and sketch the curve

2) The area of a circle is increasing at the rate of $4cm^2/s$, find the rate of change of the circumference when the radius is 6cm

GENERAL EVALUATION

1) A curve is defined by $f(x) = x^3 -6x^2-15-1$ find (i) the derivative of f(x) (ii) the gradient of the curve at the point where x= 1 (iii) the minimum and maximum points

2) The distance of a particle from a starting point is $S = t^3 – 15t^2 +63t – 40$ where t = time taken in seconds, find the (i) distance of the particle from the starting point when the particle is at rest (ii) velocity when the acceleration is zero

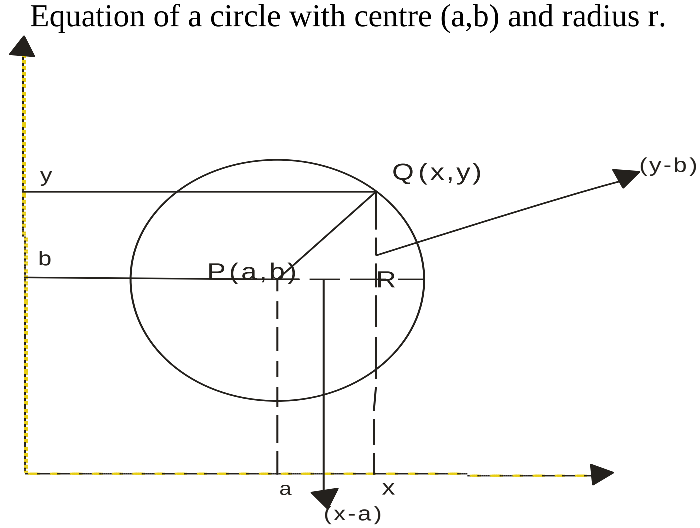
Reading Assignment : New FURTHER MATHS PROJECT 2 page 149-167

WEEKEND ASSIGNMENT

1) Find the value of x at which the function $y = x^2 – 7x^2 + 15x$ has the greatest value a) 5/3 b) 5/4 c) 5/2 d) 5/6
2) Find the values of x at the turning point of $y = 2x^3 – 3x^2 -12x + 8$ a) 1 or 2 b) -1 or -2 c) -1 or 2 d) 1 or -2
3) Find the maximum value of the function $3x^2 – x^3$ a) 2 b) 4 c) 0 d) 6
4) Find the minimum value of the function $f(x) = x^3 + 3x^2 – 9x + 1$ a) -3 b) -5 c) -4 d) 0
5) At what rate is the area of a circle changing with respect to its radius when the radius is 5cm a) 25π b 15π c) 20π d) 10π

THEORY
1) The displacement of a particle is given as $S = 12t – 15t^2 + 4t^3$ where t is the time taken . Find the velocity and acceleration of the particle after 3 seconds
2) Find the maximum and minimum points and values of the curve $y = x^3 – 6x^2 + 9x$ -

WEEK FIVE
TOPIC :THE CIRCLE: DEFITION , GENERAL EQUATION, EQUATION OF TANGENT TO CIRCLE AND LENGTH OF THE TANGENT
Definition:
A circle is defines as the locus of point equidistant from a fixed point. A circle is completely specified by the centre and the radius.
Equation of a circle with centre (a,b) and radius r.



PR = x – a, PQ=r
QR = y – b
Since $\triangle$PQR is the right angle triangle, we have:
$PQ^2 = PR^2 + QR^2$
$r^2 = (x – a)^2 + (y – b)^2$
hence, the equation of a circle with centre (a,b) and radius r is
$r^2 = (x – a)^2 + (y – b)^2$
if the centre of the circle is the origin (0,0), the equation become $x^2 + y^2 = r^2$

GENERAL EQUATION OF A CIRCLE

From $(x – a)^2 + (y – b)^2 = r^2$
$r^2 – 2ax + a^2 + y^2 – 2by + b^2 – r^2 = 0$
$x^2 + y^2 – 2ax – 2by + a^2 + b^2 – r^2 = 0$
The above equation can be written as $x^2 + y^2 + 2gx + 2fy + c = 0$
Where a = - g, b = -f, $c = - a^2 + b^2 – r^2$
Hence: $x^2 + y^2 + 2gx + 2fy + c = 0$ is called the general equation of a circle. observe the following about the general equation
i. It is a second degree equation in x and y
iiThe co-efficient of x^2 and y^2 are equal
iiiIt has no xy term

Examples:
1.Find the equation of a circle of centre (3, -2) radius 4 unit

Solution:
a = 3, b = -2 and r = 2
$(x-a)^2 + (y-b)^2 = r^2$
$(x-3)^2 + (y+2)^2 = 4^2$
$x^2 – 6x + 9 + y^2 + 4y + 4 = 16$
$x^2 + y^2 – 6x + 4y + 9 + 4 – 16 = 0$
$x^2 + y^2 – 6x + 4y – 3 = 0$
Find the centre and radius of a circle whose equation is $x^2 + y^2 – 6x + 4y – 3 = 0$

Solution:
$x^2 + y^2 – 6x + 4y – 3 = 0$
$x^2 – 6x + y^2 + 4y = + 3$
Complete the square for x and y
$x^2 – 6x + 9 + y^2\ 4y + 4 = 3 + 9 + 4$
$(x – 3)^2 + (y + 2)^2 = 16$
Compare with $(x – a)^2 + (y – b) = r^2$

Equation of a circle passing through 3 points
Find the equation of the circumcircle of the triangle whose vertices are A (2,3) B (5,4) and C (3,7)
Solution:
The equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$
$2^2 + 3^2 + 4g + 6f + c = 0$
$5^2 + 4^2 + 10g + 8f + c = 0$
$3^2 + 7^2 + 6f + 14f + c = 0$
Simplify the 3 equations
f = - 107 / 22
g = - 67 / 22
c = - 312 / 11
Hence, the equation of the circle is
$x^2 + y^2 + 2\left(\frac{-67}{22}\right) x + 2\left(\frac{-107}{22}\right) y + \frac{312}{11} = 0$
$11x^2 + 11y^2 – 67x – 107y + 312 = 0$
a = 3, b = -2, $r^2 = 16$,
$r = \sqrt{16} = 4$

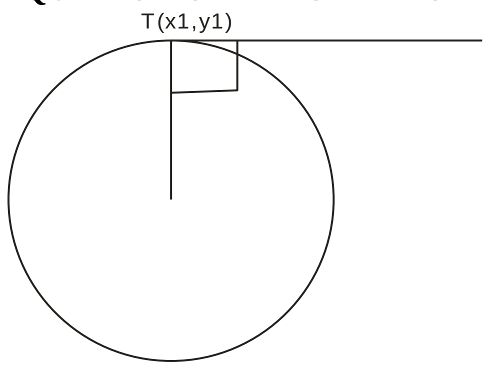
hence the centre is (3, -2) and the radius is 4 unit

Evaluation:

1.Find the equation of the circle (-1 -1) and radius 3

2.Find the centre and radius of the circle $x^2 + y^2 - 6x + 14y + 49 = 0$

C. EQUATION OF TANGENT TO A CIRCLE AT PONIT (x_1, y_1)



At x_1, y_1

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

$C = -(x_1^2 + y_1^2 + 2gx_1 + 2fy_1)$ ……………… (i)

Differentiating the equation of circle above

$2x + 2y\, dy / dx + 2g + 2f\, dy / dx = o$

Divide through bu 2

$x + y\, dy/dx + g + f\, dy/dx = 0$

$(y + f)\, dy/dx = -(x+g)$

$\frac{dy}{dx} = \frac{-(x+g)}{(y+f)}$

The equation of the tangent at x_1, y_1

$\frac{y-y1}{x-x1} = \frac{-(x+g)}{(y+f)}$

$(y - y_1)(y + f) = -(x - x_1)(x + g)$

$yy_1 + yf - y_1^2 - y_1f = -(xx_1 + xg - x_1^2 - x_1g)$

$yy_1 + yf - y_1^2 - y_1f = -(xx_1 + xg + x_1^2 + x_1g)$

$yy_1 + yf + xx_1 + xg = x_1^2 + x_1g + y_1^2 + y_1f)$

$yy_1 + + xx_1 + yf + xg = x^2 + y^2 + x_1g + y_1f)$

Adding $gx_1 + y_1f$ to both sides

$yy_1 + xx_1 + y_1f + yf + xg + gx_1 = x_1^2 + y_1^2 + x_1g + x_1g + y_1f + y_1f$

$yy_1 + xx_1 + (y_1 + y)f + (x + x_1)g = x_1^2 + y_1^2 + 2x_1g + 2y_1f$

but $x_1^2 + y_1^2 + 2x_1g + 2y_1f = -C$

$yy_1 + xx_1 + (y_1 + y)f + (x + x_1)g + C = 0$

Hence the equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + C = 0$ at (x_1, y_1) on the circle is $xx_1 + yy_1 + (x + x_1)g + (y + y_1)f + c = 0$

Examples:

Show that the point (2,3) lies on the circle $x^2 + y^2 - 3x + 4y - 19 = 0$. Hence or otherwise, determine the equation of the tangent to the circle at the point (2,3)

Solution:

$x^2 + y^2 - 3x + 4y - 19 = 0$

At (2,3)

$2^2 + 3^2 – 3(2) + 4(3) – 19 = 0$
$4 + 9 – 6 + 12 \_ 19 = 0$
R.H.S = L.H.S, hence the point (2,3) lies on the circle
$x^2 + y^2 – 3x + 4y – 19 = 0$
Compare with:
$x^2 + y^2 + 2gx + 2fy + c = 0$
$\Rightarrow$ 2g = -3, 2f = 4
$g = {-3}/{2}$ $f = {4}/{2} = 2$ c = -19
Equation of Tangent:
$yy_1 + xx_1 + (x + x_1) g + (y + y_1) f + c = 0$
$3y + 2x + (x + 2) ({-3}/{2}) + (y + 3)2 – 19 = 0$
$3y + 2x - ({3}/{2}) – 3 + 2y + 6 – 19 = 0$
$6y + 4x – 3x -6 + 4y + 12 – 38 = 0$
$10y + x – 32 = 0$
Alternatively:
$x^2 + y^2 – 3x + 4y – 19 = 0$
$2x + 2y\, dy/dx – 3 + dy/dx = 0$
$\frac{dy}{dx}\ (2y + 4) = 3 – 2x$
$\frac{dy}{dx} = \left(\frac{3-2x}{2y+4}\right)$
At 2,3

$$\frac{dy}{dx}\left(\frac{3-2\,(2)}{2(3)+\ 4}\right)\frac{-1}{10}$$

$y – y_1 = m(x – x1)$
$y – 3 = -1/10\ (x – 2)$
$10(y – 3) = -1\ (x - 2)$
$10y – 30 = -\ (x + 2)$
$10y + x – 30 – 2 = 0$
$10y + x – 32 = 0$

Evaluation
Find the equation of the tangent to the circle
1. $x^2 + y^2 + 4x – 10y – 12 = 0$ at (3,1)
2. $x^2 + y^2 – 6x – 3y = 16$ at (-2,0)

GENERAL/REVISION EVALUATION
1. Find the equation of the circle with center (1,3) and radius $\sqrt{5}$
2. Find the equation of the circle that passed through the point (0,0), (2,0) and (3, -1)
3. Find the equation of the circumcircle of the triangle whose vertices are A(2,3) B (5,4) and C(3,7)
4. Find the length of the tangent to the circle $x^2 + y^2 -2x -4y -4 =0$ from the point (8, 10)

READING ASSIGNMENT
Read equation of a circle, Further Mathematics Project II, page 205 – 210

WEEKEND ASSIGNMENT

1. What is the radius of the circle whose equation is $x^2 + y^2 - 6x - 7=0$ (a) 2 (b) 3 (c) 4 (d) 9
2. Which of the following is not an equation of a circle? (a) $x^2 + y^2 =4$ (b) $x^2 + y^2 - 2x - 3=0$ (c) $x^2 + y^2 - 2xy + 4x - 6y + 1 = 0$ (d) $2x^2 + 2y^2 - 6x + 4y + 3 = 0$
3. The equation of a circle with centre (-2, 5) and radius 3 units is (a) $x^2 + y^2 + 4x - 10y + 20 = 0$ (b) $x^2 + y^2 + 4x - 10y + 26 = 0$ (c) $x^2 + y^2 + 4x - 10y - 38 = 0$ (d) $x^2 + y^2 + 4x - 10y + 39 = 0$
4. Find the coordinates of the centre of the circle $2x^2 + 2y^2 - 4x + 12y - 7 = 0$ is (a) (-1, 3) (b) 1, 3) (c) (2, -6) (d) 41 (e) 10
5. The equation of a circle of radius 3 is $x^2 + y^2 + 10x - 8y + k = 0$. Find the value of the constant K (a) -50 (b) 18 (c) 32 (d) 41 (e) 10

THEORY

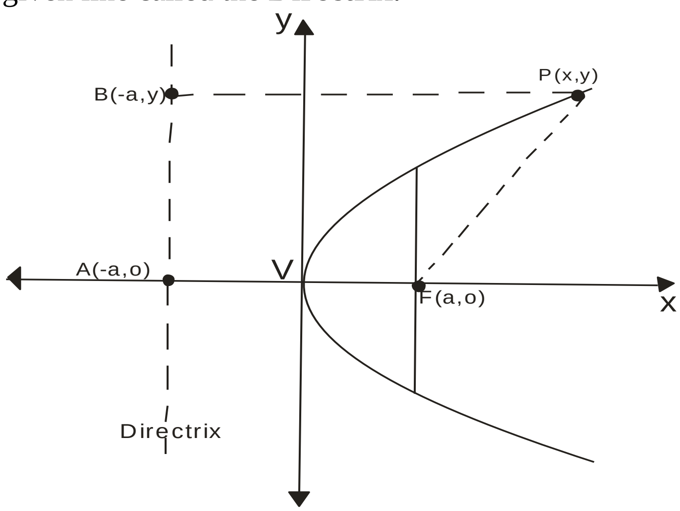
1. The equation of a circle is $x^2 + y^2 - 10x + 8y = 0$ find (i) its radius (ii) its area
2. A circle passes through the points (0,3) and (4,1), if the centre of the circle is on the x – axis, find the equation of the circle.

WEEK SIX

CONIC SECTIONS: PARABOLA,ELLIPSE AND HYPERBOLA

THE PARABOLA

The parabola is a locus of points, equidistant from a given point, called the **Focus** and from a given line called the **Directrix.**



(Length of directrix from V, (AV) = Length of Focus from V ,(FV))

The line AB, a distance of***a,*** from the y axis is called the **Directrix.** The line AF is called the axis of ***symmetry.***

Since

$BP = FP$

$BP^2 = FP^2$

$(x + a)^2 = (x-a)^2 + (y-0)^2$

$x^2 + 2ax + a^2 = x^2 - 2ax + a^2 + y^2$

$4ax = y^2$

thus, $\mathbf{y^2 = 4ax}$ is the equation of the parabola.

The line RQ which is perpendicular to AF is called the **latusrectum, V** is called the vertex and F the **focus** of theparabola.

If the vertex of the parabola is translated to a point \((x_1,y_1), the equation of the parabola becomes $\mathbf{(y-y_1)^2 = 4a(x-x_1)^2}$**.**

The above equation is said to be in the **standard** or **canonical**form

Examples

1. find the focus and directrix of the parabola $y^2 = 16x$
2. write down the equation of the parabola $y^2- 4y-12x+40 = 0$ in its canonical form and hence find i) the vertex; ii) the focus; iii) the directrix of the parabola

` **Solution**

1. compare$y^2 = 16x$ with $y^2 = 4ax$,

$4a = 16$, $a = 4$

Thus the **focus** is (4,0) while the **directrix** is $x = -4$

2. $y^2- 4y-12x + 40 = 0$

$y^2- 4y-+4-12x+40 = 0+4$ …. (***completing the square***)

$y^2- 4y+4 = 12x - 36$ …… (***rearranging***)

$(y - 2)^2 = 12(x-3)$ …… (***factorising***)

But $(y – y_1)^2 = 12(x-x_1)$

i) hence vertex $(x_1,y_1) = (3,2)$

ii) since $4a = 12$, $a = 3$ then the focus $(x_1+a,y_1) = (3+3,2) = (6,2)$

iii)the equation of the directrix is $x = 3-3$ ie $x=0$

note that the the**directrix** is of equal but opposite distance from the **vertex** with the**focus** this means the distance between the **focus** and the **vertex** = the distance between the **directrix**and the **vertex**

Equation of Tangent and Normal at point (x_1,y_1) to a Parabola

1. Equation of Tangent

$$\frac{y-y1}{x-x1} = \frac{2a}{y1}$$

2. Equation of Tangent

$$\frac{y-y1}{x-x1} = -\frac{y1}{2a}$$

Example: find the equation of the tangent and normal to a parabola

i) $y^2 = 12x$ at point (3,6)

ii) $y^2 = 16x$ at point (1, -4)

Solution

ii)$y^2 = 16x$, compare with $y^2 = 4ax$, thus a= 4

equation of tangent

$$\frac{y-y1}{x-x1} = \frac{2a}{y1} \text{ ‘ } \frac{y--4}{x-1} = \frac{2(4)}{-4}$$

Thus

y + 2x +2 =0 is the equation of the tangent

equation of the normal

$\frac{y-y1}{x-x1}$ = - $\frac{y1}{2a}$, $\frac{y--4}{x-1}$ = - $\frac{-4}{2(4)}$

Thus

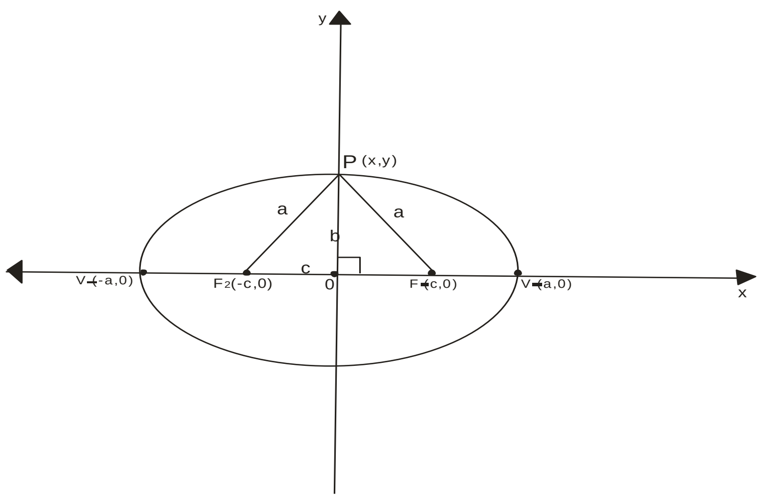
2y –x + 9 = 0 is the equation of the normal

Evaluation

1. Find the foci and directrices of the following Parabolae
   (a) $y^2 = 32x$ (b) $x^2 = 12y$
2. Write the equation of the parabola, $y^2 - 6y - 2x + 19 = 0$ in the canonical form hence determine
   Its vertex and focus

THE ELLIPSE

An **ellipse** is the locus of a point P, moving in a plane such that the sum of its distances from two fixed points F_1 and F_2 called the **foci**, is a constant**.**



OV = PF =a, OP = b and OF = c

Where ***V*** is the ***vertex*** or ***vertices***, and ***F*** is the ***focus*** or ***foci .***

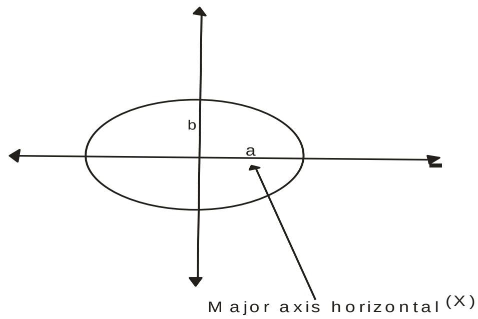
The equation of an ellipse is given by

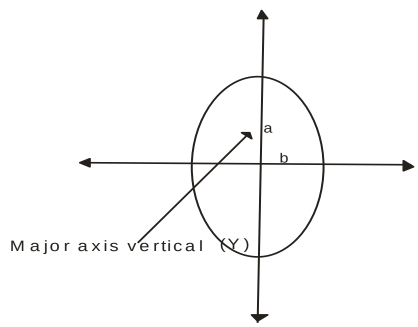
$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 or $\frac{(x-x')^2}{a^2} + \frac{(y-y')^2}{b^2}$ = 1 with centre (x^1 , y^1) (a>b) ***major axis on x……eqn(i)***

$\frac{x^2}{b^2} + \frac{y^2}{a^2}$ = 1 or $\frac{(x-x')^2}{b^2} + \frac{(y-y')^2}{a^2}$ = 1 with centre (x^1 , y^1) (a>b) ***major axis on y……eqn(ii)***

$\mathbf{a^2 = b^2 + c^2}$

where***a*** and ***b*** are on the major and minor axis respectively.





Examples

1. Find the foci and four vertices of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$

2. Write down the equation of the ellipse $25x^2 + 4y^2 - 50x - 16y - 59 = 0$ in the canonical form and hence find

i) the coordinates of the centre of the ellipse

ii) the four vertices of the ellipse

iii) the two foci of the ellipse

Solution

1. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Since a>b , then

$\frac{x^2}{9} + \frac{y^2}{25} = \frac{x^2}{b^2} + \frac{y^2}{a^2}$

By inspection,

$a^2=25$ and $b^2=9$ thus a=+5 or -5 , b= +3 or -3 and c = $+\sqrt{21}$ or $-\sqrt{21}$

$c^2=a^2-b^2$, c= +4 0r -4

i) the foci $f(0,c) = f_1(0,4)$ and $f_2(0,-4)$

ii) the vertices $V(a,0) = V_1(3,0)$ and $V_2(-3,0)$

the co-vertices $V(0,b) = V_3(o,5)$ and $V_4(o,-5)$

2. $25x^2 + 4y^2 - 50x - 16y - 59 = 0$

$25x^2 - 50x + 4y^2 - 16y = 59$

$25(x^2 - 2x + 1 - 1) + 4(y^2 - 16y + 4 - 4) = 59$

$25(x^2 - 2x + 1) - 25 + 4(y^2 - 16y + 4) - 16 = 59$

$25(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 100$

$25(x-1)^2 + 4(y-2)^2 = 100$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{25} = 1$

But $\frac{(x-x\prime)^2}{b^2} + \frac{(y-y\prime)^2}{a^2} = 1$

By inspection, a = +5 or -5 and b = +2 or -2

i) the coordinates of centre = (1,2)

ii) the vertices of the vertical axis are

$V(0+x_1, a+y_1) = V_1(1,7)$

$V(0+x_1, -a+y_1) = V_2(1,-3)$

vertices of the vertical axis are
$V(b+x_1 , 0 + y_1) = V_3(3,2)$
$V(-b+x_1 , 0+y_1) = V_4(-1,2)$
iii) the two foci are
$F(0+x_1,c+y_1) = F_1(1 , \sqrt{21} +2)$
$F(0+x_1,-c+y_1) = F_1(1 , -\sqrt{21} +2)$

Equation of Tangent and Normal at (x_1 , y_1) to an Ellipse

$\frac{y-y1}{x-x1} = \frac{-b^2x1}{a^2y1}$ is the equation of the tangent

$\frac{y-y1}{x-x1} = \frac{a^2y1}{b^2x1}$ is the equation of the normal

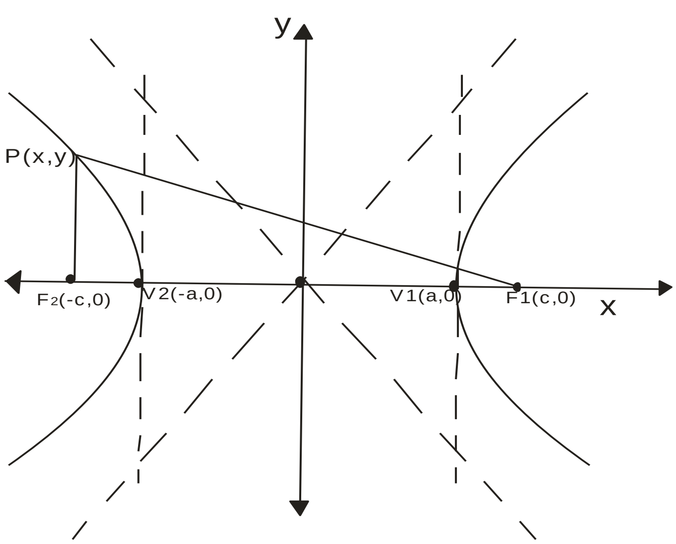
Example: find the equation of the tangent and normal to the ellipse $4x^2 + 25y^2 = 100$

Evaluation

1. Find the foci and vertices of the following ellipses
   (a) $9x^2 + 10y^2 = 90$ (b) $4y^2 + 5x^2 = 20$
2. Write the equation of the ellipse, $4x^2 + 5y^2 - 24x - 20y + 36 = 0$ in the canonical form hence determine
   Its vertices and foci

THE HYPERBOLA

The hyperbola is the locus of a point ***P***, moving in a plane such that the distance from two fixed points called the ***foci*** have a constant difference



The equation of an ellipse is given by

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{(x-x')^2}{a^2} - \frac{(y-y')^2}{b^2} = 1$ where $b^2 = c^2 - a^2$

The equation of a Tangent to an ellipse at point (x_1 , y_1) is given by

$\frac{y-y1}{x-x1} = \frac{b^2x1}{a^2y1}$

The equation of a Normal to an ellipse at point (x_1 , y_1) is given by

$\frac{y-y1}{x-x1} = \frac{-a^2y1}{b^2x1}$

Examples

1. Find the vertices and foci of the Hyperbola $25x^2 - 16y^2 = 400$
2. Find the equation of the tangent and normal to the Hyperbola $9x^2 - 36y^2 = 36$

Solution

1. $25x^2 - 16y^2 = 400$

$\frac{x^2}{16} - \frac{y^2}{25} = 1$ ***(in the canonical form)***

Since $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then

a = +4 or -4 b = +5 or -5 c = $+\sqrt{41}$ or $-\sqrt{41}$ hence

the vertices are $V_1(4,0)$ and $V_2(-4,0)$

the foci are $F(\sqrt{41}, 0)$ and $F(-\sqrt{41}, 0)$

The General Conic

A ***conic***in general may be defined as the locus of a moving point ***P,*** such that its distance fixed Point called the ***focus***, and its distance from a fixed line called the ***directrix*** are in constant ratio. This constant ratio is called the ***eccentricity*** of the conic denoted by ***e***. for

(i) a parabola $e = 1$
(ii) an ellipse $e < 1$
(iii) a hyperbola $e > 1$

GENERAL EVALUATION

1. Write the equation of the ellipse, $x^2 + 3y^2 + 2x - 24y + 46 = 0$ in the canonical form hence determine. Its vertices and foci
2. Find the vertices and foci of the hyperbola $25x^2 - 4y^2 = 100$
3. Find the equation of the tangent and normal to the parabola $y^2 - 18x = 0$ at point (2,6)

READING ASSIGNMENT: *New Further Mathematics Project 3 by TuttuhAdegun and Godspower5th Edition*

WEEKEND ASSIGNMENT

1. Find the equation of the tangent to the ellipse $4x^2 + 9y^2 = 36$ at point (0,-2)
   A, 6y + x = 9 B, 3y + x = 9 C, 6y + 4x = 9 D, y +6 x = -9
2. Find the foci of the ellipse $4x^2 + 9y^2 = 72$
   A, 14 or -14 B, $\sqrt{14}$ or $-\sqrt{14}$ C, 5 or -5 D, $\sqrt{7}$ or $-\sqrt{7}$
3. Find the equation of the normal to the parabola y^2-20x = 0 at (3 , $2\sqrt{15}$)
   A. $5y + \sqrt{15}x = 13\sqrt{15}$ B. 3y + x = 9 C. $5y + x = 13\sqrt{15}$ D. 5y + 15 x = 13
4. Which of the following is true eccentricity ***e*** of a parabola A.e< 1 B. e> 1 C. e = 1 D. e >= 1
5. Which of the following is true eccentricity ***e*** of a hyperbola A. e < 1 B. e > 1 C. e = 1 D. e >=1

THEORY

1. (a) Show that the points Q(6 , 2) lies on a circle $x^2 + y^2 - 4x + 2y - 20 = 0$ lies on a circle
   (b) Find the equation of the tangent to the circle at the point Q
2. write the equation of the following ellipses in their canonical form and hence determine

Their foci and vertices

(a) $4x^2 + 5y^2 - 24x - 20y + 36 = 0$

(b) $4x^2 + 6y^2 - 24x + 60y + 162 = 0$

WEEK SEVEN

Revision of weeks one to six

1. Differentiation: Limits of Function and First Principle, Differentiation of Polynomial
2. Differentiation of Transcendents: Derivative of Trigonometric Functions and Exponential Functions
3. Differentiation (Continued): Rules of Differentiation Product rule, quotient rule.
4. Application of differentiation rate of change, equation of motion maximum and minimum points and values of functions.
5. Conic Sections: Equation of circles, General equation of circles, finding centre and radius. Equation and length of tangents to a circle.
6. Conic Sections: The Parabola, Hyperbola and Ellipse

WEEK EIGHT

TOPIC:PROBABILITY: SAMPLE SPACE,EVENT SPACE ,INDEPENDENT AND DEPENDENT EVENTS.

Definition:

Probability: Is a measure of the likelihood that an event will occur in any one trial. Probability can be applied in several areas like insurance, industrial quality control and so on.

It could also be defined as the ratio of required outcome to the total outcome.

Probability = $\dfrac{\textbf{No of required outcome}}{\textbf{Total outcome}}$

Sample Space: This is the set of all possible outcomes of any random experiment, and it's denoted by S. The number of outcomes is denoted by n(S).

Event Space: This is the collection of outcomes of a random experiment. The number is denoted by n(E).

Outcome: This is result of an experiment in probability.

The probability that an event is certain to happen is 1, while the probability that an event is certain not to happen is zero (0).

Range of inequality: $0 \le pr(E) \le 1$

Hence; Prob (an event will occur) + prob (an event will not occur) = 1.

EXAMPLE:

1. From a box containing 2 red,6 white and 5 black balls, a ball is randomly selected . What is the Probability that the selected ball is (i) black (ii) white (iii) not black?

Solution:

Sample space = 2+ 6 + 5 = 13

n(red) = 2, n(white) = 6, n(black) = 5

(i) Prob. (black) = 5/13 (ii) Prob. (white) = 6/13

(iii) Prob. (not black) = 1 – prob. (black)

(iv) = 1 – 5/13 = 8/13

2. What is the probability that an integer selected from the set of integers {20, 21, …, 30} is a prime number?

Evaluation: 1. A box contains five 10 ohm resistors and twelve 30 ohm resistor. The resistors are all unmarked and of the same physical size. If one resistor is picked out at random, determine the probability of its resistance being 10ohms.

EQUIPROBABLE SAMPLE SPACE:

Coin: A coin has two faces called the head (H) and the tail (T). The outcome of the experiment involving a coin depends on the numbers of trials.

In a single throw of a fair coin: {H, t} = 2

Two coins thrown once or a coin thrown twice: {HH, HT, TH, TT} = 4

Three coins of thrown thrice: {HHH, HHT, HTT, HTH, THH, THT, TTH, TTT} = 8.

Example: In a single throw of two fair coins. Find the probability that : (i) two tails appear (ii) one head one tail appear (iii) two heads appear (iv) one tail one head in that order.

Solution:

S= **{HH, HT, TH, TT} = 4**

(i) Pr (two tails) = {TT} = ¼
(ii) Pr (=one head one tail) = {HT, TH} = 2/4 = 1/2
(iii) Pr (two heads) = {HH} = ¼
(iv) Pr (1 head 1 tail in that order) = (HT) = 1/4

Die: A fair die is a six faced die. A die could be tossed in a different number of trials.

When a die is tossed once: {1, 2, 3, 4, 5, 6} = 6

When a die is tossed twice or 2 dice tossed, the outcome represented in the table below and the total outcome is 36.

	1	*2*	*3*	*4*	*5*	*6*
1	***1,1***	***1,2***	***1,3***	***1,4***	***1,5***	***1,6***
2	***2,1***	***2,2***	***2,3***	***2,4***	***2,5***	***2,6***
3	***3,1***	***3,2***	***3,3***	***3,4***	***3,5***	***3,6***
4	***4,1***	***4,2***	***4,3***	***4,4***	***4,5***	***4,6***
5	***5,1***	***5,2***	***5,3***	***5,4***	***5,5***	***5,6***
6	***6,1***	***6,2***	***6,3***	***6,4***	***6,5***	***6,6***

total outcome = 36.

Example:

1. If two fair dice are tossed together. What is the probability that the total score will be: (i) ten (ii) at least 4 (iii) a prime number?

Solution:

	1	*2*	*3*	*4*	*5*	*6*
1	***1,1***	***1,2***	***1,3***	***1,4***	***1,5***	***1,6***
2	***2,1***	***2,2***	***2,3***	***2,4***	***2,5***	***2,6***
3	***3,1***	***3,2***	***3,3***	***3,4***	***3,5***	***3,6***
4	***4,1***	***4,2***	***4,3***	***4,4***	***4,5***	***4,6***
5	***5,1***	***5,2***	***5,3***	***5,4***	***5,5***	***5,6***
6	***6,1***	***6,2***	***6,3***	***6,4***	***6,5***	***6,6***

total outcome = 36.

(i) (Total score of 10) = (4,6), (5,5), (6,4) = 3
Pr (total score of 10) = 3/36 = 1/12
(ii) (at least 4) = (1,3),(1,4),(1,5),(1,6),(2,2),(2,3)……… = 33
Pr(total of at least 4) = 33/36 = 11/12
(iii)(total score of a prime number) =
(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5), =15
Pr(total score of a prime number) = 15/36 =5/12

EVALUATION*:*
1. Two fair dice are tossed. Find the probability (a) of not getting a total of 9 (iii) that the two dice show the same number.
2. In a single throw of three fair coins, find the probability that: (a) one head two tails appear (b) at least one head appears.

MUTUALLY EXCLUSIVE EVENTS: Two events are mutually exclusive if they cannot occur at the same time. That is no common element between them. This leads to the **ADDITION RULE.**
Addition rule: :If E_1, E_2, E_3, ...are dependent events than $Pr(E_1 UE_2 .... UE_n) = Pr (E_1) + Pr (E_2) + ... Pr(E_n)$
Words such as; or, either are used to indicate addition of Probabilities.
Example: In a single throw of a fair die, what is the probability that an even number or a perfect square greater than 1 shows up?
Solution: S = {1, 2, 3, 4, 5, 6} = 6
Even number = {2, 4, 6} = 3, perfect square > 1 = {4}
Pr (even nos) = 3/6 Pr (Perfect square> 1) = 1/6
Pr (even of perfect square) = 3/6 + 1/6
= 2/3

INDEPENDENT AND DEPENDENT EVENTS
Independent event: Tow or more events are said to be independent when the occurrence of one event does not affect the occurrence of the other events in any way. Hence, the events can occur independently. E .g obtaining a 6 in a single throw of a die and obtaining a tail in an event of coin. This leads to the **MULTIPLICATION RULE.**
Multiplication rule:If $E_1, E_2, E_3, ...E_n$ **are independent events then $Pr(E_1 n E_2 … n E_n)$=Pr(E1) x Pr(E2) x …Pr (En)**
When two events are combined with words such as; **AND, BOTH.** The probabilities of the events are multiplied.
An event is said to independent when picking is done with replacement.
Dependent Event: Two of more events are dependent when the occurrence of event 1 affects the occurrence of the other event (s). It is dependent when it is done without replacement.
Conditional Probability: This is the application of the multiplication rule.

Example:

1. A class consists of 8 men and 7 ladies. Two students were selected randomly to represent the class in a debate. Find the probability that two students selected are (a) both ladies (b) both men (c) of the same sex (d) of different sexes.

   Solution:

   Total students = 8+ 7 =15

   n(men) = 8, n(ladies) = 7

   a) Pr(both ladies) = 7/15 x 6/14 =1/5
   b) Pr(both men) = 8/15 x 7/14 =4/15
   c) Pr(same sex) = Pr(m & m) or Pr (L & L)

      4/15 + 1/5 = 7/15
   d) Pr (different sexes) = P (m & L) or P (L or m)

      = (8/15 x7/14) + (7/15 x 8/14)

      = 4/15 + 4/15 = 8/15

2. Two marbles are selected randomly, from a box containing 5 red, 7 white and 8 blue marbles one after the other without replacement. Find the Probability that: (a) both are red (b) one is white and the other is blue. (c0 both are of the same colour.

   Solution:

   Total Marbles = 5 + 7 + 8 = 20

   N(R) = 5, n(w) =7, n(B) = 8

   (a) Pr (both red) = P(RR) = 5/20 x 4/19 =1/19

   (b) Pr (one white, one blue) = The arrangement is important.

   Pr(wB) or P(Bw) = (7/20 x 8/19) + (8/20 x 7/19)

   = 14/95 + 14/95 = 28/95

   (c) Pr(same colour) = Pr(RR) or Pr(WW) or Pr (BB)

   = (5/20 x 4/19) + (7/20 x 6/19) + (8/20 x 7/19)

   = 20/380 + 42/380 + 56/380

   = 118/380 =59/180

EVALUATION*:*

1. A box contains 2 white and 3 blue identical marbles. If two marbles are picked at random, one after the other, without replacement, what is the probability of picking two marbles of different colour?
2. A ball is picked at random from a bag containing 5 green balls, 3 white balls and 2 black balls. What is the probability that it is either green or black?

FURTHER EXAMPLE:

1. Two gubernational aspirants A and B in two different states of Nigeria have probabilities 2/9 and 4/11 respectively of winning in an impending election. Fid the probability that: (a) both of them win in their respective states; (b) both of them lose in their respective states; (c) at least one of them wins in his state.

   Solution:

   Aspirants: A B

   Pr (A wins) + pr (A losing) = 1, Pr(Bwins) + Pr(B losing) = 1

   Pr (A losing) = 1 – 2/9 Pr (B losing) = 1- 4/11

=7/9 **7/11**

(a) **Pr**(both winning) =2/9 x 4/11
= **8 /99**

(b) Pr(both winning) = 7/9 x 7/11 = 49/99

(c) Pr (at least one wins) = Pr (A wins & B loses) or Pr(Bwins&Aloses) or Pr (A wins & B wins)
= (2/9 x 7/11) + (4/11 x 7/9) + (2/9 x 4/11)
= 11/99 + 28/99 + 8/99 = 50/99.

Evaluation: The probabilities that Bala and Uzoamaka will pass an examination are given as 0.8 and 0.9 respectively. Find the probability that: (a) both of them fail the examination (b0 at least one of them will pass this examination.

GENERAL EVALUATION/REVISIONAL QUESTIONS

1. The probabilities that Ade and Bayo passed a exam are 2/3 ad 3/5 respectively find the probability that (i) two of them passed (ii) two of them failed.
2. A six- sided die is thrown once. What is the probability that the result will be (a) an even number (b) a number less than 6 (c) a number greater than 2?
3. A universal set had 24 elements and A and B are subsets of the universal set such that n (A) = 14, n(B) =9 and n(A∩ B) = 6. If P (a) is the probability of selecting an element belonging to set A, calculate (a) P (A) (b) P (A∩B) (c) P (B^1) (d) P$(AU B)^1$

Reading Assignment ; New Further Maths Project 2 page 198 – 210

WEEKEND ASSIGNMENT

1) In a single throw of a fair coin ,find the probability that a head appears a) ¾ b) ½ c) 2/3 d)¼
Two fair dice are tossed , find the probability that the total score is
2) prime number a) 5/12 b) ¾ c) ½ d) 5/36
3) less than 6 a) 5/36 b) 5/24 c) 5/6 d) 5/18
A bag contains 6 red and 4 blue identical marbles, if two marbles are picked one after the other without replacement, find the probability that both marbles are of
4) the same colour a) 8/15 b) 7/15 c) 1/3 d) 4/15
5) differentcolour a) 4/15 b) 7/15 c) 8/15 d) 4/9

THEORY

1) Two dice are thrown together, what is the probability of getting (i) at least 6 (ii) score greater than 8
2) A box contains 5 white and 3 black balls , if two balls are drawn one after the other with replacement what is the probability that both of them are (i) of the same colour (ii) of different colour

WEEK NINE

TOPIC:PERMUTATION AND COMBINATION :

PERMUTATION:Definitoin, Concept , Different Arrangement Of Items, Cyclic Permutation .

A. Definition, Concept:

Definition: Permutation is defined as the number of arranged of objects. The different orders of arrangement are important. E.g. Find the number of ways of arranging the letters pqr.

Pqr, prq, qrp, qpr, rqp. The number of ways is 6 ways
Similarly, for 4 letters the number of arrangement is 24
In general, the number of different arrangement of n different objects is equal to n! (n factorial)
N! = n x (n-1) x (n-2) x … x 3x2 x 1x0! (But, 0! = 1)
1.Simplify the following: A. 5! B. $\frac{7!}{3!\ 4!}$

Solution
a.5! = 5x4x3x2x1 = 120
b.$\frac{7!}{3!\ 4!}$ = 7x6x5x4! = 7x5 = 35
2.Find the number of ways of arranging the letters of the word MACHINE

Solution:
There are seven different letters in the word MACHINE, therefore the number of permutation is 7! = 7x6x5x4x3x2x1 = 5040 ways
3.Simplify $\frac{(n+1)!}{(n-1)!} = \frac{(n+1)n!}{(n-1)n!} = \frac{n+1}{n-1}$

ARRANGEMENT OF n-OBJECTS TAKING r-OBJECTS
If we are interested in the number of ways 2 letters of a 4 lettered word cnbe arranged, then the np_r is the permutation of n objects taking at a time
$^np_r = \frac{n!}{(n-r)!}$
Example: Evaluate: (a) $8p_3$ (b) $11p_9$
Solution:
(a) $^8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!!} = \frac{8x7x6x5!}{5!}$ = 8x7x6 = 336
(b) $^{11}P_9 = \frac{11!}{(11-9)!}$ = 11x0x9x8x7x6x5x4x3x2! = 19958400
4.In how many ways can three people be seated on eight seats in a row?
Solution:
1st seat can be occupied by any of the 8 = 8 ways
2nd seat can be occupied in 6 ways
Hence, the number of ways = 8x7x6 = 336 ways
Alternatively, n = 8, r = 3
$^nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8x7x6x5!}{5!}$ = 8x7x6 = 336 ways

EVALUATION
1.In how may ways can 8 students be seated in a row?
2.In how many ways can the 1st, 2nd 3rd prizes be won by 6 athletetes in a race?
3.In how many ways can the letters of the word HISTORY be arranged?

CYCLIC PERMUTATION: Cyclic permutation is the arrangement of things around a circular object. Since a circular table has no beginning and no end, the number of arrangement is 1 x (n – 1) !
If the circular object can be turned over e.g. circular ring e.t.c. the number of arrangement = $\frac{(n-1)!}{2}$

Example: In how many ways can 6 members of a disciplinary committee be seated round a circular table?

Solution:
The number of ways = (n – 1)! X 1
N = 6,
Hence, (6 – 1)! X 1 = 5! X 1 = 120 ways
PERMUTATION OF IDENTICAL OBJECTS:
The number of ways of permuting n objects taking n at a time with n, objects alike, $n_2$alke is,

$$\frac{n!}{N1!\ \ N2!\ \ N3!\dots nj!}$$

$$examples:$$

1. Find the number of ways the word MATHEMATICS can be arranged.
Solution:

MATHEMATICS
There are: 2Ms, 2Asm 2Ts and 11 letters.
N=11, n1 = 2! N2 = 2! N3 = 2
$\frac{11!}{2!2!2!}$ = 11x10x9x7x6x5x4x3x2x1 = 4989600

CONDITIONAL PERMUTATION:
Sometimes restrictions are placed on the order of arrangements of objects
Examples:
1.Find the number of ways the letters of the word COMMITTEE can be permuted, if the 2Ts must always be together.
Solution:
The 2Ts must be together, we can lump them as follows: COMMI (TT) EE = 8!
=
$\frac{8x7x6x5x4x3x2x1}{2!\ 2!}$= 10080 ways
2.Find the number of ways of arranging the letters of the word MOSHOESHOE if the letter M must always begin a word
Solution:
Since letter m must always begin, and then m can only occupy the first position
i.e M = 1 way
other letters, OSHOESHOE = $\frac{9!}{3!\ 2!\ 2!\ \ 2!}$ = 9x7x6x5x4 = 7560 ways

COMBINATION: Selection, Conditional Selection And Its Application
Combination can be defined as the number of ways r – objects can be selected from n – objects irrespective of the arrangement
Hence, the notation is thus, ${}^nC_r$ or $({}^n{}_r)$
, ${}^nC_r$= $\frac{n!}{(n-r)!r!}$
Relationship between permutation and combination is thus, nCr = $\frac{nPr}{r!}$
Example:

1. Evaluate $^{10}C_4$

Solution:

$^{10}C_4 = \frac{10!}{(10-4)!\ 4!}$ = 10 x 3 x 7 = 210

2. In how many ways can three books be selected from 12 books?

SOLUTION:

N = 12, r = 3, $^{12}C_3 = \frac{12}{(12-3)!3!}$ = 2x11x10 = 220 ways

3. A committee consisting of 3 men and 5 women is selected from 5 men and 10 women. Find how many ways this committee can be formed.

Solution:

MEN
R = 3, n = 5
$^5C_3 = \frac{5!}{(5-3)!3!} = 10$

WOMEN
r = 5, n = 10
$^{10}C_5 = \frac{10!}{(10-5)!5!} = 252$

Therefore number of ways of selecting the committee = 10x252 = 2520 ways.

GENERAL/ REVISION EVALUATION

1.Find the number of ways the letters of the word FURTHER can be arranged
2.Find the number of ways of arranging 7 people in a straight line, if two particular people must always be separated
3.In how many ways can 6 pupils be lined up if 3 of them insist in the following one another
4.Verify that $\frac{(n-3)!}{n!}$ = (n – 1) (n – 2) (n – 3)!

READING ASSIGNMENT

Read permutation and combination, further mathematics project 2 pages 47-54

WEEKEND ASSIGNMENT

1. Evaluate $^6C_2 + {^6C_3} + {^6C_4} + {^6C_5}$ (a) $^6C_6$ (b) $^6C_5$ (c) $^8C_5$
2. How much ways can the letters of the word EVALUATE be arranged? (a) 10080 (b) 20160 (c) 40320
3. In how many ways can 2 boys and 3 girls be arranged to sit in a row, if the boys must sit together (a) 6 (b) 4 (c) 24
4. Find the number of ways 6 people can be seated in a round table, if two particular friends must sit next to each other (a) 48 9b) 24 (c) 120
5. In how many ways can 6 pupils be lined up if 3 of them insist on following one another? (a) 720 (b) 144 (c) 24

THEORY

1. Out of 7 lawyers, 5 judges, a committee consisting of 3 lawyers, 2 judges is to be formed, in how many ways can this be done, if
   a. Any lawyer and any judge can be included
   b. One particular judge can be included
   c. Two particular lawyer cannot be in committee
2. If $^nP_3 / {^nC_2} = 6$, find the value of n

WEEK TEN

TOPIC:DYNAMICS :NEWTON'S LAWS OF MOTION, MOTION ALONG INCLINED PLANE AND MOTION OF CONNECTED PARTICLES

Sir Isaac Newton put forward three important laws which relate to the motion of bodies under the action of given forces. These laws are central to the study of dynamics, since **dynamics** essentially involves the study of motion of bodies under given forces

The Frist Law OF Motion

If we place a ball on the ground, it will continue to rest there until someone comes to kick it. Once it is kicked, it will start moving and continue to move until something happens either to stop it or change its direction of motion. This basic idea is stated in Newton's First Law which may be stated as: **Everybody continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external impressed forces.**

This law re-emphasizes the fact that a force can change the state of rest or uniform motion of a body. A stationary point will remain stationary unless it is pushed from its stationary position. By pushing, we are exerting a force on the object. A moving car will continue to move unless brakes are applied to bring it to a halt. The brakes applied have introduced a kind of force that makes the car to come to a stop.

The tendency of a body to remain in its state of rest or uniform motion in a straight line is called **inertia** and is a function of the mass of a body. The greater the mass of a body, the greater its inertia and hence the greater the force required to change the state of the body.

The Second Law of Motion

The law states that **the rate of change of momentum of a body is proportional to the applied force and is in the direction of the force.**

The second law of motion helps us to obtain an expression for 5the force acting on a body. We recall that momentum is defined as the product of mass and velocity.

By the second law of Newton

$F \ \alpha \frac{d}{dt}(mv)$

Since the mass of a given body is a constant, we have

$F \ \alpha \ m\frac{dv}{dt}$

$\therefore F \quad = km\frac{dv}{dt}$

Where k is a constant.

By a suitable choice of unit for F, we can make

k =1

Hence

$F = m\frac{dv}{dt}$. But $\frac{dv}{dt}$ = a

$\therefore \boldsymbol{F = ma}$

Where **a** is the acceleration of the body.

The law established an exact relationship between force F, the mass m of a body, as well as the acceleration **a** of the body.

If a force Facts on a body of mass m kg it produces an acceleration in the mass given by the relation

F = ma

Newton's second law also enables us to deduce the unit of force. We recall that the unit of mass is kilogramme (kg). The unit of mass is meter per second (ms^{-2}). Hence, the unit of force is kg ms^{-2}.

A force acting on a body of mass 1 kg, producing an acceleration of 1 ms^{-2} is called **1 Newton (1N).** So the unit of force is the Newton.

Newton's Third Law of Motion

Newton's third law of motion states: **Action and reaction are equal and opposite.** When two bodies are in contact, the forces of action and reaction are equal in magnitude and opposite in direction.

Such forces are also collinear. Let us consider a heavy block placed on a table, the force due to gravity on the body (weight of the block) acts directly on the table downwards. The table will have to exert an equal but opposite force on the block. This force acts upwards and balances the weight of the block on the table. If the table cannot withstand the weight of the block, it collapses.

Example 1

A boy sits on a log. The mass of the log is 8 kg and the weight of the boy is 55N. What is the reaction of the ground on the log on which the boy is sitting? (Take g= 9.8ms^{-2})

Solution

Weight of the log = 8 × 9.8N

=78.4N

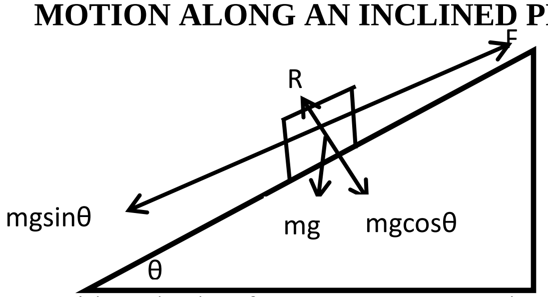
Weight of the boy and the log = (78.4 + 55) N

= 133.4N

By the third law of Newton, the ground will expert an equal but opposite force on the log on which the boy is sitting.

Hence if R is the force reaction of the

MOTION ALONG AN INCLINED PLANE



Consider a body of mass m on a smooth plane inclined at angle θ to the horizontal.

The force on the body due to gravity (weight) acts vertically downward and is mg.The force which acts perpendicularly to the inclined plane in mg cosθ .

The reaction of the inclined surface on the body is R and is equal in magnitude to mg. The force which tends to move the body down the plane is mgsinθ. The force which tends to move the body up the plane is F –mg sinθ. The equation of motion is:

F – mgsinθ = ma

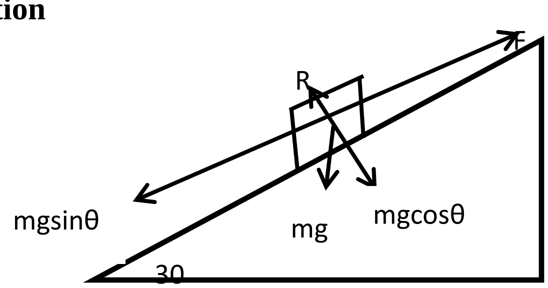
Where a is the acceleration of the body. If however, F <mgsinθ then the body will move down the plane with a net force of mgsinθ – F = ma where a is the acceleration of the body down the plane.

Example 5

An object whose weight is 10kg is placed on a smooth plane inclined at 30 to the horizontal. Find:

a) the acceleration of the object as it moves down the plane;
b) the velocity attained after 3 seconds if:
   (i) it starts from rest;
   (ii) it moves with an initial velocity of 5ms^{-1}
   [Take g = 10 ms^{-2}

Solution



The force acting on the body down the plane is mg sin 30^0

The force acting onm the body up the plane is Zero. The net force acting on the body down the plane is mg sin 30^0

From newton's law

Mg sin 30^0 = m×a

Where a is the acceleration of the body down the plane.

10 × 10 × sin 30^0 = 10×a

50 = 10a

a = 5 ms^{-2}

(b) (i) if the body start from rest, u = 0.

from equations of motion

v = u + at

= 0 +5 × 3

= 15 ms^{-1}

(iii) if the body moves with an intial velocity of 5 ms^{-1}, then

v = u+at

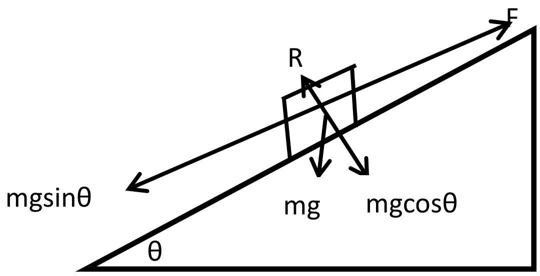
=5+3×5

= (5 +15)ms^{-1}

Example 6

A body of mass m is placed on the surface of a smooth plane which is inclined at an angle θ to the horizontal. A force f whose line of action is parallel to the surface of the inclined plane acts on the body to just prevent it from slipping down the plane. If R is the reaction between the surface of the inclined plane and the body, show that F = R tanθ

Solution



Since the body lies on the surface of the plane

$R - mgcos\theta = 0$, $R = mgcos\theta$….(1)

The force which tends to move the body down the plane is mg sinθ

Since the force F just prevents the body from slipping down the plane

$F = mg\ sin\theta$

Dividing (2) by (1)

$\frac{F}{R} = \frac{mg\ sin\theta}{mg\ cos\theta}$

$= \frac{sin\theta}{cos\theta}$

$\frac{F}{R} = tan\theta$ Hence, F = R tanθ

MOTION OF CONNECTED PARTICLES

In this unit, we shall examine the motion of two or more bodies connected by a light inextensible string, connected to a light smooth pulley. The basic assumption we make is that the tensile force in the string is always the same throughout every section of the string. We can easily write down the equation of motion of the connected particles, once they are set in motion.

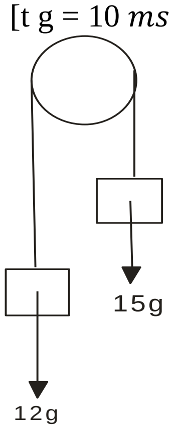
Example

The particles whose masses are 15kg and 12 kg respectively are attached to the ends of a light inextensible string. The string passes over a light frictionless pulley and the masses hang freely. The system is released from rest when the 15kg mass is 32m above the floor. Find:

(i) the tension in the string;

(ii) the time taken by the 15kg mass to reach the floor.

[t g = 10 ms^{-2}]



Let a be the acceleration , for the 15kg mass

The net force is 15g –T

15g –T =15a

For the 12kg mass

The net force is T-12kg
By Newton"s 2nd law T-12kg
3g = 27a a =g/9a = 10/9 , T – 12g = 12a
T = 12a + 12g =12x 10/9 +12 x10 =400/3

EVALUATION
A force P acts on a body of mass 5kg on a smooth horizontal floor if it produces an acceleration of 4.5 m/s , find the magnitude of P

GENERAL EVALUATION
1) A body of mass 15kg is placed on a smooth plane which is inclined at 60 to the horizontal, find the acceleration of the body as it moves down the plane
2) A body of mass 5kg is connected by a light inelastic string which is passed over a fixed frictionless pulley by a movable frictionless pulley of mass 1kg over which is wrapped another light inelastic string which connects masses 3kg and 2kg , find the acceleration of the masses and the tension in the strings

Reading Assignment
New Further Maths Project 2 page 237- 242

WEEKEND ASSIGNMENT
A body of mass of mass 100kg is placed in a lift , find the reaction between the floor of the lift and the body when the lift moves upward
1)at constant velocity a) 800N b) 900N c) 1000N d) 600N
2) with an acceleration of 3.5m/s a) 100N b) 1350N c) 1200N d) 1500N
3) A body of mass 20kg is placed in a lift , find the reaction between the floor of the lift and the body when the lift moves downward with a retardation of 2.5 m/s a) 250N b) 300N c) 350N d) 400N
4) Law of inertia is also known as Newton"s ----- Law of motion a) 2nd b) 1st c) 3rd d) 4th
5) The relationship between force and acceleration of a body in motion can be attributed to Newton"s -------- Law of motion a) 1st b) 2nd c) 3rd d) 4th

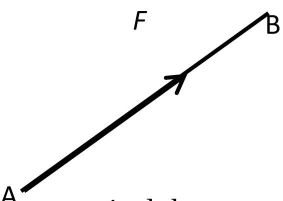
THEORY
1) A car of mass 0.9 tonnes is moved by a constant force F from a speed of 12m/s to 16m/s over a distance of 50m, find F
2) Two masses 10kg and 8kg are connected by a light inextensible string which is passed over a light frictionless pulley fin the tension in the string

WEEK ELEVEN
TOPIC:WORK, POWER AND ENERGY ;IMPULSE AND MOMENTUM
Work
If the point of application of a force is displaced, the force is said to do work. For a constant force *F*, whose point of application is given a displacement *d*, work done is the product of */F/* and *d*.

The **work** done by a constant force is defined as the product of the force and the distance moved by its point of application along the line of application of the force.

W= / *f*/*d*

Consider a force *F* displaced a distance d along is line of application *AB.*



If the magnitude of the displacement is d then

W= /*F*/ × d

Power

Power is the rate at which work is being done. For example, if work of 90j is done in 15 seconds, the power is 6J/sec. The unit of power is the **Watt** (W)

Example 16

On the level, a car develops a power of 60KW. if the resistance to motion is 9ooN, what is the maximum speed of the car?

Working at the same power and with the same resistance operating, what would be the maximum speed possible up an inclined plane whose slope is $\sin^{-1}\left(\frac{1}{50}\right)$, if the mass of the car is 800kg?

What is the acceleration at the time when the car is moving up the inclined plane at $40ms^{-1}$? (Take g= $10ms^{-2}$)

Solution

Let *P* be the power developed by the car.

Let *W* be the work done.

Let *F* be the tractive force of the car

$P = \frac{d}{dt}(w)$

$=\frac{d}{dt}(f\times s)$

Energy

Kinetic Energy

The work done in bringing a particle of mass m from rest to a velocity v is called the kinetic energy of the particle

If we donate E_k as the kinetic of particle of mass m reaching a velocity v from rest, then

$E_k = \frac{1}{2}mv^2$

If *W* is the work done in bringing the particle of mass *m* to a velocity v from an initial velocity *u*, and if *s* is the distance travelled in the process, then

$W = F\times s$

$= m \times a \times s$, where *a* is the acceleration

But

$s = \frac{(u+v)}{2}t$

$a = \frac{(v-u)}{t}$

$\therefore W = m \times a \times s$
$= m \times \left(\frac{v-u}{t}\right) \times \left(\frac{v+u}{2}\right)t$
$=\frac{1}{2}mv^2 - \frac{1}{2}mu^2$
If we denote the last expression bΔ
$\Delta E_k = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
$\Delta E_k$is the change in kinetic energy in bringing a particle of mass *m* from an initial velocity *u* to final velocity *v*

Potential energy
Potential energy of a particle of mass *m* is the energy of the particle, by virtue of its position relative to a reference level. It is the work done in bringing a particle from a reference level to a height *h*
If we donate the potential energy by E_k then
$E_p = F \times s$
Since F = *mg*, and S = /
E_p= mgh

Law of Conversation of Energy
This is a generalization if the experience from nature and it state that energy cannot be created nor destroyed. It can only be changed from one from to another
When a particle falls from a height, it loses potential energy. This loss in potential energy is compensated for, in the gain in kinetic energy
Example
A particle starting from rest falls freely from a height *H* above the ground. If *g* is the acceleration due to gravity, show from energy consideration that the velocity *v* with which the particle strikes the ground is given by the expression
$v = \sqrt{2gH}$
Solution
Let the particle have mass *mkg.*
Loss of potential energy = *mgH.*
Gain in kinetic energy = $\frac{1}{2}mv^2$
from the principle of conversation of energy:
Impulse and Momentum
We recall from Newton's secondary law of motion that for a constant force *F*
$F = m\frac{dv}{dt}$
or F dt = m dv
The expression on the L.H.S. of (1) gives the time- effect of force and is called Impuse. The expression on the R.H.S gives the velocity effect of mass and is called change in momentum. The expression (1) is a restatement of newton's second law of motion ant it states that **the time-effect of force is equal to the change in momentum**.
From Newton's second law of motion
$F = m\frac{dv}{dt} = ma$

GENERAL EVALUATION

1) A body of mass 20kg moves a distance of 8m in the direction of line of action of force F =5N on a smooth table find the work done
2) A particle of mass 3kg is projected vertically upward with an initial velocity of 5 m/s from the ground, calculate the potential energy at the greatest height
3) A sphere of mass 12kg and another sphere of 8kg moves toward each other with velocities 5 m/s and 3 m/s respectively find the speed of the sphere after collision
4) Calculate the loss in kinetic energy caused by the collision of the two bodies above in (3)

Reading Assignment

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WEEKEND ASSIGNMENT

A body at rest and of mass 8kg is acted upon by a force of 30N for 0.4 seconds, calculate the
1) impulse on the body a) 120Ns b) 240Ns c) 3.2Ns d) 12Ns
2) final speed of the body a) 1.5m/s b) 2.5m/s c) 2.0m/s d) 3m/s
3) distance covered within the time interval a) 3m b) 30m c) 0.3m d) 0.354m
4) kinetic energy possessed by the body a) 6J b) 9J c) 12J d) 15J
5) power of the body a) 50W b) 22.5W c) 45W d) 12.5W

THEORY

1) A body of mass 6kg moves with speed 3m/s , if it is acted upon by a force of 18N for 4 seconds find the speed of the body.
2) The resistance to the motion of a cart being pushed by a man is 220N, if the man pushed the cart a distance of 10km for 45 mins calculate (i) the work done by the man (ii) power exerted by the man.